National Academy of Sciences of Ukraine

Institute of Mathematics

International Conference

DYNAMICAL SYSTEMS AND THEIR APPLICATIONS

Kyiv, Ukraine

June 22 – 26, 2015

Abstracts

Kyiv – 2015

DYNAMICAL SYSTEMS
AND
THEIR APPLICATIONS

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International Conference on Dynamical Systems and Their Applications is organized by the *Institute of Mathematics, National Academy of Sciences of Ukraine.*

Dynamical systems theory, being one of the rapidly developing areas of modern mathematics, provides powerful theoretical base for exploring a variety of models that arise in natural and social sciences, engineering and technology. The combination of the internal wealth and beauty of results with the exceptional practical importance motivates a growing number of specialists to study into dynamical systems.

Beginning with the 60s, the Institute of Mathematics held conferences and schools on various fields of mathematics, in particular, on dynamical systems. This has had a profound effect on the development not only of dynamical systems theory but also of the overall nonlinear dynamics. Not so long, the Institute of Mathematics decided to arrange International Conference "Dynamical Systems and Their Applications" (ICDSA), aimed to promote transnational cooperation and share good practice in the field of dynamical systems theory. The first conference hosted in Kyiv in 2012. The second edition of ICDSA takes place in Kyiv again, it considers a wide range of issues of the modern theory of dynamical systems.

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DYNAMICS OF THE THUE-MORSE SYSTEM OF DIFFERENCE EQUATIONS

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The Thue–Morse system of difference equations was introduced in [1] as a model to understand the electric behavior (conductor or insulator) of an array of electrical punctual positive charges occupying positions following a one dimensional distribution of points called a *Thue–Morse chain* which it is connected to the sequence $\mathbf{t} = (0110100110010\ldots)$ called also the *Thue–Morse sequence*. Unfolding the system of difference equations, we obtain the two-dimensional dynamical system in the plane given by

$$F(x, y) = (x(4 - x - y), xy).$$

The interest of such system was stated by A. Sharkovskĭi as an open problem and proposing some questions.

The most interesting dynamics of the system is developed inside an invariant plane triangle, where hyperbolic periodic points of almost all period appear, there are subsets of transitivity and invariant curves of spiral form around the unique inside fixed point.

In this talk we will present some results concerning the behavior of all points outside the triangle, completing the known dynamics of the system. In fact we have obtained that outside the triangle, the orbits of all points are unbounded. Some of them go to infinite in an oscillating way occupying the second and third quadrant of the plane and others are going in a monotone way to infinite. Outside the triangle there are no periodic points. Such new results has an interesting interpretation in terms of the physics of the problem. Additionally we will answer some of the questions stated by Sharkovskĭi concerning the inside of the mentioned triangle.

We will also present graphycal analysis of such evolutions and also the visualization of the dynamics of the system inside the triangle.

Additionally we will present results on another system associated to Fibonacci sequence whose unfolding in $\mathbf{R}^3$ is

$$F(x, y, z) = (y, z, yx - z).$$

[1] Y. Avishai and D. Berend, *Transmission through a Thue–Morse chain*, Physical Review B **45**(6) (2011), 2717–2724.

ON LIMIT CYCLE BIFURCATIONS

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We carry out the qualitative analysis of polynomial dynamical systems. To control all of their limit cycle bifurcations, especially, bifurcations of multiple limit cycles, it is necessary to know the properties and combine the effects of all of their rotation parameters. It can be done by means of the development of new bifurcational and topological methods based on the well-known Weierstrass preparation theorem and the Perko planar termination principle stating that the maximal one-parameter family of multiple limit cycles terminates either at a singular point which is typically of the same multiplicity (cyclicity) or on a separatrix cycle which is also typically of the same multiplicity (cyclicity) [1].

If we do not know the cyclicity of the termination points, then, applying canonical systems with field rotation parameters, we use geometric properties of the spirals filling the interior and exterior domains of limit cycles. Using this method, we have solved, e. g., the problem of the maximum number of limit cycles surrounding a singular point for an arbitrary polynomial system and Hilbert's Sixteenth Problem for a general Liénard polynomial system with an arbitrary (but finite) number of singular points [2]. Applying a similar approach, we have completed the strange attractor bifurcation scenario which connects globally the homoclinic, period-doubling, Andronov–Shilnikov, and period-halving bifurcations in the classical Lorenz system [3]. We discuss also how to apply this approach for studying global limit cycle bifurcations of discrete polynomial (and rational) dynamical systems which model the population dynamics in biomedical and ecological systems.

This work was partially supported by the Simons Foundation of the International Mathematical Union and the Department of Mathematics and Statistics of the Missouri University of Science and Technology.

[1] V. A. Gaiko, *Global Bifurcation Theory and Hilbert's Sixteenth Problem*. Kluwer, Boston (2003).

[2] V. A. Gaiko, *The applied geometry of a general Liénard polynomial system*. Appl. Math. Letters **25** (2012), 2327–2331.

[3] V. A. Gaiko, *Global bifurcation analysis of the Lorenz system*. J. Nonlinear Sci. Appl. **7** (2014), 429–434.

ABSOLUTELY NON FREE ACTIONS, RANDOM SUBGROUPS, AND FACTOR REPRESENTATIONS

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I will present results of several investigations, performed in collaboration with M. Benli, L. Bowen, A. Dudko, R. Kravchenko and T. Nagnibeda. These results deal with invariant and characteristic random subgroups in some groups of geometric origin, including hyperbolic groups, mapping class groups, groups of intermediate growth and branch groups. During the talk the role of totally non free actions will be explained. This will be used to explain why branch groups have infinitely many factor representations of type II_1.

TOWARDS THE GLOBAL BIFURCATION THEORY ON THE PLANE

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Local bifurcation theory (in what follows we will talk about the plane only) is related to transfigurations of phase portraits of differential equations. Currently this theory is almost completed. Nonlocal theory is related to bifurcations of separatrix polygons (polycycles). Though in the last 30 years there were obtained many new results, this theory is far from being completed. Recently it was discovered that nonlocal theory contains another substantial part: a global theory. New phenomena are related with appearance of the so called sparkling saddle connections. The aim of the talk is to explain first results of the new theory and discuss numerous open problems.

REGULAR AND CHAOTIC OSCILLATIONS IN SINGULARLY PERTURBED SYSTEMS WITH DELAY

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The article offers special research methods for investigating both local and nonlocal dynamics of wide range singularly perturbed systems with delay. When studying local behaviour of solutions it appeared to be characteristic a realization of critical cases of the problem regarding station stability of infinite dimensions. Available methods of invariant integral manifolds and methods of normal forms tend to be not applicable.

The author has developed a special method of quasinormal forms based on arranging special series of nonlinear evolutionary equations which do not contain small or big parameters, and nonlocal dynamics of which describes local behaviour of solutions for an original system with delay.

An efficient method implying reduction to finite-dimensional mapping is proposed to investigate nonlocal dynamics of singularly perturbed systems with delay. The dynamics of the latter describes the structure of original system attractors. The result is asymptomics of both regular and irregular relaxation oscillations. Applications have been considered. A number of conclusions have been made on dynamic features specific exclusively for systems with delay.

ON FLEXIBILITY OF ENTROPIES AND LYAPUNOV EXPONENTS

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Dedicated to the memory of Dmitry Viktorovich Anosov and Nikolai Chernov

Principal stochastic properties of conservative dynamical systems come in two varieties: (i) asymptotic distribution of orbits, that includes ergodicity, various kinds of mixing properties, decay of correlations and so on, and (ii) global measures of complexity and speed of divergence, i.e. measure theoretic (Kolmogorov) entropy, topological entropy, as well as Lyapunov exponents with respect to the absolutely continuous invariant measures and measures of maximal entropy. Classical work of Anosov from 1960s and more recent work of Chernov are among the inspirations of the program I will outline on this talk.

Another motivation for the program comes form various recent results concerning smooth actions of higher-rank abelian groups as well as the older Zimmer program concerning actions of "large" non-abelian groups. Among rigidity phenomena established for those actions are strong restrictions of arithmetic nature on the values of Lyapunov exponents for positive entropy invariant measures as well as on values of topological and measure-theoretic entropies.

On the other hand, for classical rank one systems,, i.e diffeomorphisms and flows, one does not expect any restrictions of similar nature. Surprisingly though this is easier said than done. There is a variety of local results coming from various constructions of Anosov and non-uniformly hyperbolic systems but already such a simply sounding question as characterizing all possible triples of positive numbers that appear as Lyapunov exponents of volume-preserving Anosov diffeomorphisms (w.r. to the volume measure) on the three-dimensional torus seems to require some serious new ideas beyond combination of known methods.

So far, non-trivial progress was made in a joint work with Alena Erchenko in the characterization of pairs of numbers that appear as values of topological and measure-theoretic (Liouville) entropies for the geodesic flow on a surface of genus greater than one with a Riemannian metric of a fixed total area.

TOPOLOGICALLY-METRICAL AND FRACTAL ANALYSIS OF LOCAL STRUCTURE OF CONTINUOUS FUNCTIONS

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Now fractal properties of functions are studied in various directions. In particular,

(1) properties of level sets of function and sets of its peculiarities;
(2) property of function to preserve or transform fractal dimension, frequencies of digits, mean value of digits, etc.;
(3) self-similar, self-affine and auto-modeling properties of functions and their graphs;
(4) properties of dynamics generated by functions with fractal properties;
(5) construction of functions with given fractal properties;
(6) development of suitable and effective tools for definition and investigation of functions with complicated local structure (using various systems of encoding of real numbers with finite, infinite, constant and variable alphabet); by means of systems of functional equations and iterated functions.

In the talk, we study some finite- and infinite-parameter families of functions in above-mentioned directions.

We consider classic binary representation of real numbers:

$$[0;1) \ni x = \sum_{n=1}^{\infty} \alpha_n 2^{-n} \equiv \Delta^2_{\alpha_1\alpha_2\ldots\alpha_n\ldots},$$

where $\alpha_n \in A_2 \equiv \{0,1\}$ is an alphabet of binary numeral system, as well as its reencoding by means of infinite alphabet $A = Z_0 = \{0,1,2\ldots\}$:

$$\overline{\Delta}^2_{a_1a_2\ldots a_n\ldots} \equiv \Delta^2_{\underbrace{1\ldots1}_{a_1}0\underbrace{1\ldots1}_{a_2}0\ldots\underbrace{1\ldots1}_{a_n}0\ldots} = \Delta^2_{\alpha_1\alpha_2\ldots\alpha_n\ldots}, \quad a_n \in A.$$

The last is called $\overline{\Delta}^2$-representation. It has extra-zero redundancy, i.e., every number has a unique $\overline{\Delta}^2$-representation.

We describe fractal properties of dynamical system with phase space $[0,1] \subset R^1$ and mapping

$$f(x) = f(\overline{\Delta}^2_{a_1(x)a_2(x)\ldots a_n(x)\ldots}) = \overline{\Delta}^2_{\varphi(a_1,a_2)\varphi(a_2,a_3)\ldots\varphi(a_{n-1},a_n)\varphi(a_n,a_{n+1})\ldots},$$

where φ is a function of two variables defined on $Z_0 \times Z_0$ and taking the values from the alphabet Z_0. The simplest examples are $\varphi(a_1,a_2) = a_1a_2$ or $\varphi(a_1,a_2) = a_1 + a_2$.

ω-ATTRACTORS AND THEIR BASINS

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Dynamical systems on a compact space X generated by a continuous map $f : X \to X$ are considered. The asymptotic behavior of every trajectory $f^i(x)$, $i = 0, 1, 2, ...$, is usually characterized by its ω-limit set $\mathcal{A}_x = \bigcap_{m>0} \overline{\{\bigcup_{i>m} f^i(x)\}}$. Each of the sets $\mathcal{A}_x$, $x \in X$, can be an ω-limit set for many other trajectories. It therefore makes sense to refer to each set, which is an ω-limit set at least for one trajectory, as an ω-attractor. As is well known, if a dynamical system has so-called Smale's horseshoe, then it has a lot of ω-attractors. In particular, if X is an interval I and the map f has a cycle of period $\neq 2^k$, $k \geq 0$, then, for certain $m > 0$, the map f^m has a one-dimensional horseshoe (or Λ-scheme). Namely, there are two disjoint intervals J_1, J_2 such that $f^m(J_1), f^m(J_2) \supset J_1 \cup J_2$, and, as a consequence, the dynamical system has continuum many different ω-attractors. The talk deals with the properties of the dynamical system on an ω-attractor and properties of the set of all ω-attractors of the dynamical system considered as a set in the space 2^X of all closed subsets of X endowed with the Hausdorff metric.

The set of trajectories, which are attracted by an ω-attractor, is called the basin of the ω-attractor: If $\mathcal{A}$ is an ω-attractor, then $\mathfrak{B}(\mathcal{A}) = \{x \in X \mid \mathcal{A}(x) = \mathcal{A}\}$ is a basin of this ω-attractor. In the talk, we discuss the structure of the basins of ω-attractors, as well as some unresolved problems. It is known that $\mathfrak{B}(\mathcal{A})$ is always an $F_{\sigma\delta}$-set, i.e., it is represented in the form $\bigcap_k \bigcup_j F_{jk}$ with F_{jk} being closed sets, but this upper estimate of the basin complexity is achieved even in the case $X = I$, and $\mathfrak{B}(\mathcal{A})$ is a set of the third Baire class in this case.

Most of the results presented in the talk has been obtained in the 60s of the last century, but are not widely known until now yet.

σ-ATTRACTORS, μ-ATTRACTORS, AND THEIR BASINS

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If we have to account the statistical behavior of the trajectory of a point x, we consider its σ-limit set $\mathcal{A}_\sigma x$, so called "statistically limit set" of the trajectory, i.e., the smallest closed set such that $\lim\limits_{N\to\infty} \frac{1}{N}\sum_{i=0}^{N-1}\chi_U(f^i(x)) = 1$. It is the smallest closed set satisfying the property that any neighborhood of this set contains *almost all* points of the trajectory. As a rule, there are a lot of trajectories that have the same σ-limit set, so we consider the structure of the set of all points that have the same σ-limit set. This set is the basin of the corresponding attractor.

If a σ-attractor is not just one periodic trajectory, then trajectories of the dynamical system on such an attractor can generate many (several, finitely many, or continuum) different invariant measures, even if the attractor consists of only two fixed points. For a probability measure μ, which is defined on Borel subsets of a compact space X and invariant with respect to a continuous map $f : X \to X$, we consider the set $\mathcal{B}(\mu) = \Big\{x \in X : \mathcal{C}_n\varphi(x) \to \int \varphi\, d\mu$ for all $\varphi \in C(X)\Big\}$, where $\mathcal{C}_n\varphi(x) = \frac{1}{n}\sum_{i=0}^{n-1}\varphi(f^i(x))$ are Cesaro means for the trajectory of x and $C(X)$ is the space of continuous functions on X. It is the set of trajectories that generate the measure μ because $\mathcal{B}(\mu)$ can be defined as the set of points x such that $\mathcal{C}_n\delta_x$, where δ_x is the probability measure concentrated at x, converges weakly to μ. Also, along with $\mathcal{B}(\mu)$, we consider the set $\mathcal{D}(\mu)$ that consists of the points x, for which μ belongs to the set of limit points of the sequence $\mathcal{C}_n\delta_x$.

In the theory of dynamical systems, along with open and closed invariant sets, there appear also F_σ sets (e.g., the set of all periodic points), G_δ sets (e.g., the set of all orbitally stable points), $F_{\sigma\delta}$ sets, etc. Sometimes, instead of this Hausdorff classification, the Bair (Lusin–de la Valée Poussin) classification is used. In this classification, the first class includes all sets which are both F_σ and G_δ; the second class consists of the sets which are either F_σ or G_δ but not both and of sets which are $F_{\sigma\delta}$ and $G_{\delta\sigma}$ at the same time but do not belong to the first class; etc. Usually upper descriptive estimates for dynamical systems are obtained easily. It is much harder to prove that these upper estimates can be reached. In our talk, we will give descr0iptive estimates for basins of attractors commonly used in the theory of dynamical systems and formulate some proven results and hypothesis on the exactness of the stated estimates.

FRACTALS, SINGULAR PROBABILITY MEASURES, AND DYNAMICAL SYSTEMS

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It is well known now that fractal analysis plays an important role in the study of chaotic dynamical systems and singular probability distributions [4]. Rather often fractals appear as attractors resp. repellers of corresponding dynamics or spectra resp. minimal dimensional supports of singular measures. On the other hand methods of dynamical systems are pointed out to be extremely useful to study fine fractal properties of sets as well as probability measures supported by fractals [1, 4, 5].

During the talk we shall discuss several new phenomena related to interplays of the theory of transformations preserving the Hausdorff dimension [3], faithfulness resp. non-faithfulness of fibred systems [2, 5], infinite IFS and fine fractal properties of singular probability distributions [1, 2].

[1] S. Albeverio, Yu. Kondratiev, R. Nikiforov, G. Torbin, *On fractal properties of non-normal numbers with respect to Rényi f-expansions generated by piecewise linear functions.* Bull. Sci. Math. **138** (2014), no. 3, 440 – 455.

[2] S. Albeverio, Yu. Kondratiev, R. Nikiforov, G. Torbin, *On new fractal phenomena connected with infinite linear IFS.* submitted to Acta Mathematica, 2015.

[3] S. Albeverio, M. Pratsiovytyi, G. Torbin, *Fractal probability distributions and transformations preserving the Hausdorff-Besicovitch dimension.* Ergodic Theory and Dynamical Systems, **24** (2004), no.1, 1–16.

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[5] F. Schweiger, *Ergodic theory of fibred systems and metric number theory.* Oxford University Press, New York, (1995).

NEW INVARIANTS OF TOPOLOGICAL CONJUGACY OF NON-INVERTIBLE INNER MAPPINGS

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Let $f : X \to X$ be an inner surjective map of a locally compact locally connected metric space X. Recall that an inner map is an open and isolated map. A map is open if the image of an open set is open. A map is isolated if the pre-image of a point consists of isolated points.

The book [1] and some earlier papers introduced a set of new invariants of topological conjugacy of non-invertible inner mappings that are modeled from the invariant sets of dynamical systems generated by homeomorphisms. Those new invariants are based on the analogy between the trajectories of a homeomorphism and the directions in the set of points having common image which is viewed as having 2 dimensions.

In particular, this papers introduced the sets of neutrally recurrent and the neutrally non-wandering points related to the dynamics of points and neighborhoods in that "extra" dimension. Those invariants provide a natural language for the topological classification of many classes of polynomial maps and also allow to define analogs of many well known classes of invertible maps such as Smale diffeomorphisms for the non-invertible inner maps.

[1] I. Yu. Vlasenko, *Inner mappings: topological invariants and their applications*. Inst. of Math, Kiev (2014).

ON A CLASS OF DIFFERENTIAL-FUNCTIONAL EQUATIONS

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We consider differential-functional equations with the deviation of argument depending on the unknown function, namely, the equations:

$$x'(f(x(t))) \;=\; G(x(t),\, x(f(x(t)))\,), \tag{1}$$

where $t \in \mathbb{R},\ f : \mathbb{R} \to \mathbb{R},\ G : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$. Differential-functional equation (1) is completely integrable and is reduced to one parameter family of functional equations

$$x(f(x(t))) \;=\; g(x(t), \alpha) \tag{2}$$

where α is an arbitrary constant. Equations (2) are iterative functional equations and it's investigation can be reduced to studying the so-called characteristic map

$$S : \begin{cases} t \;\mapsto\; f(x), \\ x \;\mapsto\; g(x, \alpha) \end{cases} \tag{3}$$

generated by (2). This allows to construct general solution for such equations, [1]–[4].

We investigate properties of the equation (1) in dependence on parameters and initial conditions.

[1] M. Kuczma, *Functional equations in a single variable*, Warszawa, 1968.

[2] M. Kuczma, B. Choczewski, R. Ger, *Iterative functional equations*, Cambridge Univer. Press, 1990.

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[4] A. N. Sharkovsky, *On functional and functional-differential equations with the deviation of the argument depending on the unknown function*, Functional and functional-difference equations, Inst. Math., Acad. Sci. Ukrain. SSR, Kiev, 1974, pp. 148–155 [in Russian].

THE COST OF APPROXIMATE CONTROLLABILITY AND A UNIQUE CONTINUATION RESULT AT INITIAL TIME FOR THE GINZBURG-LANDAU EQUATION

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We consider the controlled Ginzburg-Landau equation with an internal distributed control in a sub-domain. The complex Ginzburg-Landau equation describes the evolution of a complex-valued field $y = y(x,t)$ by

$$\begin{cases} \partial_t y - (a+ib)\Delta y = Ry - (\alpha + i\beta)\left|y\right|^2 y + \chi_\omega u, & \text{where } t>0,\ x \in \Omega \subset \mathbb{R}^N, \\ y = 0 & \text{on } \partial\Omega \times (0,T), \\ y(x,0) = y_0(x) & \text{in } \Omega. \end{cases} \tag{1}$$

Here a, b, R, α and β are some positive real numbers.

The fundamental technique approached in this paper is estimating Carleman type inequalities for the adjoint linearized system. We renew the computations made by Rosier and Zhang in [4], and obtain explicit coefficients in the Carleman estimates, with respect to T, where $[0,T]$ is the maximum interval of time we consider. We obtain explicit bounds of the cost of approximate controllability, i.e., of the minimal norm of a control needed to control the system approximately.

Given $y_0 \in L^2(\Omega)$, a final state $y_1 \in L^2(\Omega)$ and $\varepsilon > 0$, there exists a control $u \in L^2(\omega \times (0,T))$ such that the solution of (1) satisfies

$$\left\|y(T) - y_1\right\|_{L^2(\Omega)} \leq \varepsilon.$$

As in [3], the approximate control u of minimal norm in $L^2\left(\omega \times (0,T)\right)$ corresponding to $y_0 = 0$, $y_1 \in L^2\left(\Omega\right)$ and $\varepsilon > 0$ can be obtained by minimizing the convex functional J in $L^2\left(\Omega\right)$:

$$J(p_T) = 1/2 \iint\limits_{\omega\times(0,T)} \left|p\right|^2 dxdt + \varepsilon \left\|p_T\right\|_{L^2} - \int\limits_{\Omega} y_1 p_T dx.$$

We prove an unique continuation result at initial time, which relies on Carleman estimates with explicit coefficients. In [1], the authors establish an unique continuation result at initial time for a second-order parabolic operator P in $[0,T] \times \mathbb{R}^N$, $P = \partial_t + A.P = \partial_t + A$, where A is a second-order elliptic operator. In [2], Lefter and Lorenzi force the local result in [1], to a nonlocal one. The authors are interested when the unique continuation is global at initial time, i.e., when y solves the homogeneous parabolic equation in $L^2\left(0,T;L^2\left(\Omega\right)\right)$ and $\omega \subset\subset \Omega$ is an open subset, under which conditions on the behavior of $\left\|y\right\|_{L^2(0,t;H^1(\omega))}$, when $t \longrightarrow 0$ one obtains that $y\left(x,0\right) = 0$ for all $x \in \Omega$.

We want to establish an unique continuation result at the initial time for the operator $P = \partial_t - (a+ib)\Delta$.

[1] P. Albano, D. Tătaru, *Unique continuation for second-order parabolic operators at the initial time*. Proc. Amer. Math. Soc. **132(4)** (2004), 1077–1085.

[2] C. G. Lefter, A. Lorenzi, *Approximate controllability for an integro-differential control problem*. Applicable Analysis (2008).

[3] J. L. Lions, *Remarks on approximate controllability.* J. Anal. Math. **59** (1992), 103–116.

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STABILIZATION OF SYSTEMS WITH MULTIPLE POWER NONLINEARITIES

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In this paper we solve the stabilization problem for a class of nonlinear systems with uncontrollable first approximation. Namely, we consider a nonlinear system of the form

$$\begin{cases} \dot{x}_1 = u, \\ \dot{x}_i = x_{i-1}^{2k_{i-1}+1}, \quad i = 2, \dots, n \end{cases} \tag{1}$$

where $k_i \in N$, $u \in R$ is a control. The stabilization problem for system (1) is to construct a control of the form $u = u(x)$ such that equilibrium point of the closed-loop system is asymptotically stable.

We assume that $k_1 = \dots = k_s = 0$ and $0 < k_{s+1} < \ldots < k_{n-1}$ for some s such that $0 \le s \le n-2$. For $s = n-2$ the stabilization problem for system (1) has been solved in [1]. The main result of the present work states that a stabilizing control can be found in the form

$$u(x) = a_1 x_1 + a_2 x_2 + \dots + a_n x_n + \sum_{i=s+1}^{n-1} a_{n-s+i} x_i^{2k_i+1}. \tag{2}$$

The conditions on coefficients a_i are obtained with the help of the Lyapunov function method. A Lyapunov function $V(x)$ can be chosen in the following form $V(x) = (Fx, x)$ where F is a solution of a singular Lyapunov inequality.

[1] M. O. Bebiya, *Stabilization of systems with power nonlinearity*, Visn. Khark. Univ., Ser. Math, Prykl. Mat. Mekh. **1120** (2014), 85–94.

APPLICATION OF (c)-PROPERTIES FOR CESARO SUMMABILITY METHODS $(C, 1)$ OF ERGODIC THEOREM

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Let the given sequence $S = \{S_n \in B : n \in N_0\}$, where B is Banach space, $N_0 = N \cup \{0\}$. A closed convex set $G \subset B$ is the (c)-set of sequences $S = \{S_n\}$, if $\forall \varepsilon > 0\ \exists \delta(\varepsilon) > 0$ $\exists ([n_k(\varepsilon); m_k(\varepsilon)])$:

$$S_n \in G_\varepsilon \forall n \in [n_k(\varepsilon); m_k(\varepsilon)] \subset N \forall k \in N, \frac{m_k - n_k}{m_k} \geq \delta(\varepsilon),$$

where G_ε is a closed convex ε-neighborhood of a closed convex set $G \subset B$.

For the case $B = C$ of known methods Cesaro $(C, \alpha), \alpha \geq 1$, (c)-property:

If a $\exists \lim_{n\to\infty} S_n = L, (C, \alpha)$ and closed convex set the $G \subset C$ is (c)-set of sequences $S = \{S_n\} \subset C$, while $L \in G$ [1].

This property holds for the methods $(C, 1)$ in the case of Banach spaces B.

Proposition. Let $(B, \Omega, \mu)-$ the space of normalized measure and $f \in L^1(B, \Omega, \mu)$. Then for each there $x \in B_e \subset B, \mu(B \backslash B_e) = 0$ is a limit

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} f\left(T^k x\right) = \widehat{f}(x) [2, p.17 - 22],$$

that is, $\lim_{n\to\infty} f(T^n x) = \widehat{f}(x)(C, 1)$, and

1. $\widehat{f}(x) \in G_x\ \forall x \in B_e$, for which there is $(c)-$ a set $G_x \subset (B, \Omega, \mu)$ sequence $\{f(T^n x)\}$;

2. $\lim_{n\to\infty} f(T^n x) = \widehat{f}(x) \forall x \in B_e$, in which each partial sequence boundary $\{f(T^n x)\}$ is the $(c)-$ point;

3. $x \in B \backslash B_e$, if for $x \in B$ there are two different $(c)-$ sets G_1 and G_2 sequences $\{f(T^n x)\}$ such that $G_1 \subset (B, \Omega, \mu), G_2 \subset (B, \Omega, \mu)$, $G_1 \cap G_2 = \emptyset$.

The statement can be formulated for the case $f(x) = \chi_A(x)$, where $\chi_A(x)-$ the indicator set $A \subset B$.

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NONLINEAR DYNAMICS OF RANDOM LASER GENERATION IN 3D PERCOLATING CLUSTERS

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In the artificial 3D percolation medium, the clusters filled by the nanoemitters give rise to a topologically nontrivial photonic structure with no-integer fractal dimension. In such a system, the laser model is strongly modified by the spatial percolating clusters' distribution. We systematically study a random laser emission from such advanced 3D system with radiated emitters randomly incorporated in the incipient spanning cluster. The nonlinear time dynamics and spectra of the lasing output are studied numerically. To find the optimal optical path for communications between the radiated emitters the Fermat principle was applied with the use of the quantum Monte Carlo approach.

CHIMERA STATES IN A NETWORK OF OSCILLATORS UNDER CROSS-GLOBAL COUPLING

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Chimera states was surprisingly emerged in a symmetric network of identical oscillators, when the whole population splits into two subpopulations, one coherent, another noncoherent. It was first noticed [1] in phase oscillators for nonlocal coupling where the coupling has a space variation, later in limit cycle system and chaotic system. Three different characteristic features of chimera states were identified: spatio-temporal chaos [2], spatial chaos [3] and a mixture of both [4] in the noncoherent population while the coherent population may stay in periodic, chaotic state. It was also evidenced in physical experiments [5]. The strict condition of nonlocal coupling in a network was relaxed recently; it was found to emerge for linear or nonlinear global coupling [6, 7]. However, the chimera states has a characteristic spatio-temporal chaos only in the noncoherent population.

We extend [8] the work here and report chimera states in a network of identical oscillators (limit cycle and chaotic) where a linear repulsive cross-global coupling is added to the typical attractive self-global coupling. We present examples of the van der Pol oscillator and the Rössler oscillator as individual nodes of the network. Especially, in a Liénard system, we find both the spatio-temporal chaos and spatial chaos in the noncoherent population.

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THE HOMOTOPY METHOD TO SOLVE DYNAMICAL SYSTEMS AND BOUNDARY VALUE PROBLEMS FOR DIFFERENTIAL EQUATIONS

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In this paper we have discussed the analytical solution of boundary value problem

$$f''' - ff'' + 4\left(1 - f'^2\right) + M\left(1 - f'\right) = 0; \quad f(0) = K, \quad f'(0) = 0, \quad f'(\infty) = 1$$

by homotopy analysis method and also analytical solution of some nonlinear dynamical systems by homotopy analysis method.

INFINITE ORBIT EQUIVALENCE CLASS FOR A MINIMAL SUBSTITUTION DYNAMICAL SYSTEM

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The seminal paper [2] answered, among other outstanding results, the question of orbit equivalence of *uniquely ergodic* minimal homeomorphisms of a Cantor set. It was proved that two such minimal systems, (X, T) and (Y, S), are orbit equivalent if and only if *the clopen values sets* $S(\mu) = \{\mu(E) : E \text{ clopen in X}\}$ and $S(\nu) = \{\nu(F) : F \text{ clopen in Y}\}$ coincide where μ and ν are the unique invariant measures with respect to T and S, respectively. Bratteli diagrams play an extremely important role in the study of homeomorphisms of Cantor sets because any minimal (and even aperiodic) homeomorphism of a Cantor set is conjugate to the Vershik map acting on the path space of a Bratteli diagram. This realization turns out to be useful in many cases, in particular, for the study of substitution dynamical systems because the corresponding Bratteli diagrams are of the simplest form.

In this talk, we focus on the study of orbit equivalence of minimal substitution dynamical systems. For any primitive proper substitution σ, we give explicit constructions of countably many pairwise non-isomorphic substitution dynamical systems $\{(X_{\zeta_n}, T_{\zeta_n})\}_{n=1}^{\infty}$ such that they all are (strong) orbit equivalent to (X_σ, T_σ). We show that the complexity of the substitution dynamical systems $\{(X_{\zeta_n}, T_{\zeta_n})\}$ is the essential difference that prevents them from being isomorphic.

Theorem. *Let σ be a proper substitution. Then there exist countably many proper substitutions $\{\zeta_n\}_{n=1}^{\infty}$ such that (X_σ, T_σ) is orbit equivalent to $(X_{\zeta_n}, T_{\zeta_n})$, but the systems $\{(X_{\zeta_n}, T_{\zeta_n})\}_{n=1}^{\infty}$ are pairwise non-isomorphic.*

Given a primitive (not necessarily proper) substitution τ, we find a stationary simple properly ordered Bratteli diagram with the least possible number of vertices such that the corresponding Bratteli-Vershik system is orbit equivalent to (X_τ, T_τ).

The results that will be presented during the talk are published in [1].

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PROBABILITY MEASURES AND DYNAMICAL SYSTEMS GENERATED BY CONTINUOUS NOWHERE MONOTONIC FUNCTIONS

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Let p be *some prime*, $p > 2$. We consider the matrix $||d_{ij}||$ where elements:

$$d_{2kj} = p^{-1}, \quad d_{2k-1j} = -p^{-1}, \quad j = \overline{1,p}, \quad k = \overline{1,(p-1)2^{-1}}.$$

In the research we consider the function:

$$f\left(\triangle^{p^2}_{\alpha_1(x)\alpha_2(x)\ldots\alpha_n(x)}\right) = \beta_{\alpha_1} + \sum_{k-2}^{\infty}\left(\beta_{\alpha_k}\prod_{j-1}^{k-1} q_{\alpha_1}\right),$$

where

$$\beta_0 = 0, \quad \beta_k = q_0 + q_1 + \ldots + q_{k-1},$$
$$q_m = d_{[m\cdot p^{-1}],(m-p[m\cdot p^{-1}])}.$$

Theorem 1. Function $f(x)$ is correctly defined, continuous, winding and its graph is a self affinity set of the space R^2.

The report offers research results topological metric and fractal properties of these objects:

1. Distribution of function's values:

$$Y = f(X),$$

where X – random variable with predetermined distribution.

2. Dynamic system $([0;1], f, B, \lambda)$.

REDUCTION THEORY, CODING OF GEODESICS, AND CONTINUED FRACTIONS

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I will discuss a method of coding of geodesics on surfaces of constant negative curvature using boundary maps and "reduction theory". For compact surfaces these maps are generalizations of the Bowen-Series map. For the modular surface they are related to a family of (a,b)-continued fractions. In special cases, when an (a,b)-expansion has a so-called "dual", the coding sequences are obtained by juxtaposition of the boundary expansions of the fixed points, and the set of coding sequences is a countable sofic shift. I will also give a dynamical interpretation of the ”reduction theory” which underlines these constructions and its relation to the attractor of an associated natural extension map that parametrizes the corresponding cross-section of the geodesic flow. The talk is based on joint works with Ilie Ugarcovici.

ASPECTS OF THE PERMUTATION ENTROPY

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Recent results show that there is a close relation of the Kolmogorov-Sinai entropy and the relatively new concept of Permutation entropy based on measuring the diversity of ordinal patterns in a dynamical system. In order to get more insights into this relation, we discuss how the Kolmogorov-Sinai entropy of a discrete-time measure-preserving dynamical can be obtained from the ordinal patterns obtained via measurements by a collection of real-valued random variables. We show that under certain separation conditions the distribution of these patterns is sufficient for determining the Kolmogorov-Sinai entropy (see [1]). On the base of this statement, we discuss Permutation entropy and, in the case of ergodicity, the estimation of Kolmogorov-Sinai entropy. Finally, we give two new variants of Permutation entropy, a conditional and a robust one (see [1, 3]), and illustrate their performance in data analysis.

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A NOVEL MODEL FOR EXPLANATION THE REGULAR AND CHAOTIC DYNAMICS IN ARTERIAL BLOOD FLOW

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Long measurements of the blood pressure $P(t)$ and flow rate $Q(t)$ oscillations in arteries revealed prevalent regular dynamics in young healthy volunteers and frequent nonlinear chaotic dynamics in elderly [1,2]. The heart rate variability, nonlinear properties of the blood vessel wall and turbulent blood flow were discussed for explanation the chaotic behavior. As it was shown in [3], the chaotic dynamics in elderly may appear in the distant parts of the cardiovascular system (CVS), while in the central arteries the flow is quasi-regular.

In this paper the broad band noise with no distinct peaks (1/f noise) is found in the power spectrum of the $P(t)$ and $Q(t)$ time series. The maximal Lyapunov exponents are found to be negative in the central aorta and positive in the upper and lower extremities. A model of the CVS as a series connection of n viscoelastic chambers is proposed. It is shown the pressure oscillations in the chamber are governed by the nonlinear n-th order ODE

$$\sum_{j=1}^{n} A_j \frac{d^j P}{dt^J} + A_0 P = \sum_{j=1}^{n-1} B_j \frac{d^j Q}{dt^J} + A_0 Q, \tag{1}$$

where A_j, B_j are nonlinear functions of the material parameters and resistivities $\{Z_i(P_i)\}_{j=1}^{n}$ of the chambers.

Solution of (1) has been studied at wide variations of the viscoelastic properties of the chambers. It is shown the abnormal high compliance of the distant chambers may lead to varying time delays between the responds of the chambers to the pressure variations and, thus, to the chaotic dynamics. Direct applications to the medical diagnostics of deep vein thrombosis and chronic arterial insufficiency in the lower extremities are discussed.

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ON DYNAMICAL SYSTEM OF CONFLICT WITH FAIR REDISTRIBUTION OF VITAL RESOURCES

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We discuss the properties of dynamical system of conflict [1] modelling the alternative interaction between opponents.

Let the probability measures μ, ν describe the starting distribution of the vital resource space Ω for a pair of opponents A, B. The problem is to find the law of conflict interaction between A, B which ensures the compromise redistribution of Ω.

Assume that evolution changes of μ, ν are governed by the following nonlinear law of conflict dynamic (cf. with [2, 3]):

$$\dot{\mu} = \frac{\mu\Theta - \tau}{z}, \quad \dot{\nu} = \frac{\nu\Theta - \tau}{z},$$

where $\Theta = \Theta(\mu, \nu)$ is a positive quadratic form which fixes the so called conflict exponent for opponents A, B and $\tau = \tau(\mu, \nu)$ has sense of the occupation exponent. The meanings of τ at each moment of time show the values of presence of opponents A, B on the opposite territory. The denominator z ensures that measures $\mu(t), \nu(t)$ are probability for all $t > 0$.

We prove that an appropriate construction of Θ and τ ensures the existence of the ω-limit state $\{\mu^\infty, \nu^\infty\}$ which corresponds to the fair redistribution of the vital resources space Ω between opponents A, B. The fair means that μ^∞, ν^∞ coincide with the normalized components of the classic Jordan decomposition of the signed measure $\omega = \mu - \nu = \omega_+ - \omega_-$, i.e.

$$\mu^\infty = \mu_+ := \frac{\omega_+}{\omega_+(\Omega)}, \ \ \nu^\infty = \nu_- := \frac{\omega_-}{\omega_-(\Omega)}.$$

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ANALOGUE OF TOPOLOGICAL ENTROPY FOR SOME INFINITE-DIMENSIONAL SYSTEMS

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Topological entropy is one of the important invariants in the theory of dynamical systems. However, the definition of topological entropy is not effective to describe the complexity of infinite-dimensional dynamical systems. In [3], [4] it is proved that for the dynamical systems $(C([0,1],I), Z^+, T)$, where $T : C([0,1],I) \to C([0,1],I)$ has the form $T(\varphi)(t) = f(\varphi(t)),\ f \in C(I,I)$, the value of topological entropy is either 0 or $+\infty$. Thus such definition of entropy does not help us to conduct thorough analysis of such systems.

The purpose of this work is to offer an analogue of definition of entropy, by which it is possible to evaluate the complexity of such dynamical systems more effectively. By evaluating the number of (n,ε)-separated continuous functions firstly for fixed function T, and then for general case, we conclude that for systems above it is reasonable to use value $\limsup_{\varepsilon\to 0} \varepsilon \lim_{n\to\infty} \frac{\ln N(n,\varepsilon)}{n}$ as such analogue definition of entropy.

In more general case, for dynamical systems $(C(L,I),T)$, where L is a compact, $T(\varphi)(t) = f(\varphi(t))$, we can consider the value $\limsup_{\varepsilon\to 0} N_L(\varepsilon)^{-1} \lim_{n\to\infty} \frac{\ln N(n,\varepsilon)}{n}$, where $N_L(\varepsilon)$ denotes the minimal number of elements in cover of L by balls with diameter ε. This value is finite if the topological entropy of function f is finite. In the case $L \subset R^d$ this value is also nonzero if the topological entropy of f is nonzero.

If we consider system $(C(L,K),T)$, where L, K are compacts, then to evaluate its complexity we must analyze properties of sets K and L, connected with the number of elements in their ε-covers and with the structure of these ε-covers, for example, the mean dimension, considered in [1], [2].

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SOLVING FOR THE FIXED POINTS OF 3-CYCLE IN THE LOGISTIC MAP AND TOWARD REALIZING CHAOS BY THE THEOREMS OF SHARKOVSKII AND LI-YORKE

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Sharkovskii proved that, for continuous maps on intervals, the existence of 3-cycle implies the existence of all others. Li and Yorke proved that 3-cycle implies chaos. To establish a domain of uncountable cycles in the logistic map and to understand chaos in it, the fixed points of 3-cycle are obtained analytically by solving a sextic equation. At one parametric value, a fixed-point spectrum, resulted from the Sharkovskii limit, helps to realize chaos in the sense of Li and Yorke.

NUMERICAL METHODS FOR SOLVING SYSTEMS OF NONLINEAR VOLTERRA INTEGRAL EQUATIONS WITH CARDINAL SPLINES

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The aim of this paper is to develop numerical methods for solving systems of Volterra integral equations with cardinal splines. The unknown functions are expressed as a linear combination of horizontal translations of certain cardinal spline functions with small compact supports. Then a simple system of equations on the coefficients is obtained for the system of integral equations. It is relatively straight forward to solve the system of unknowns and an approximation of the original solution with high accuracy is achieved. Several cardinal splines are used in the paper to enhance the accuracy. The sufficient condition for the existence of the inverse matrix is examined and the convergence rate is investigated. Examples are given to demonstrate the benefits of the methods.

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ON DP-APPROACH TO FRACTAL PROPERTIES OF RANDOM VARIABLES WITH INDEPENDENT IDENTICALLY DISTRIBUTED LML-SYMBOLS

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The talk is devoted to the metric and dimensional theories of LML-expansions of real numbers which are generalizations of the corresponding theories of Lüroth expansions, GLS-expansions and $x - Q_\infty$-expansions.

We also develop probabilistic theory of such expansions and study fine fractal properties of the corresponding singularly continuous probability distributions.

During the talk we will discuss DP-approach for the study of properties of such distributions which is based on deep connections between transformations preserving the Hausdorff dimension and the faithfulness of Vitaly coverings.

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FORCED ROTATION OF A FERROMAGNETIC FINE PARTICLE IN A VISCOUS CARRIER: THE STATIONARY PROBABILITY DENSITY

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Ferrrofluids are the complex systems with high potential application, which are widely spread from engineering to biomedicine. Most descriptions of these media are grounded on the concept of ferrohydrodynamics [1], where ferrofluid is considered as a continuous media. But a lot of effects can be described properly only in terms of the microscopic structure of ferrofluid. In particular, the problem of the AC-field absorption and further ferrofluid heating for the case of rather fine dispersed nanoparticles was investigated within the framework of the complex magnetic susceptibility [2]. But, when the magnetic energy is comparable with the thermal one, one should to account the individual Brownian rotation of each particle.

For practical purposes, the probability density function of the nanoparticle rotational states is the main characteristic with respect to its rotational motion. The above mentioned function is the solution of the appropriate Fokker-Plank equation [3]. We suppose that the particle is under the action of the field $\mathbf{H} = h(\mathbf{e}_x \cos\Omega t + \mathbf{e}_y \sin\Omega t) + \mathbf{e}_z h_z$. Here h and Ω are the rotating field amplitude and frequency, respectively, h_z is the static field value, $\mathbf{e}_{x,y,z}$ are the unit vectors of the Cartesian coordinates, t is the time. The approximate stationary solution of the Fokker-Planck equation has the following form:

$$P(\theta,\Phi) = P_0 \cdot \left[1 - \frac{\tau^2}{2\tau_r} h\Omega \sin\theta \sin\Phi + \frac{\tau^3}{24\tau_r^2} h\Omega \left(h_z \sin 2\theta \sin\Phi + h \sin^2\theta \sin 2\Phi\right)\right], \qquad (1)$$

where $P_0 = C \sin\theta \exp\left[-W\tau/\tau_r\right], W = -h\sin\theta\cos\varphi - h_z\cos\theta, C$ is the normalization constant, θ, φ are the angular coordinates of the nanoparticle magnetic moment, $\Phi = \varphi - \Omega t$, $\tau_r = 6\eta/M^2, \tau = 6\eta V/k_{\mathrm{B}}T$, η is the liquid viscosity, M is the nanoparticle magnetisation, V is the nanoparticle volume, k_{B} is the Boltzmann constant, T is the temperature. Our analytical findings were confirmed by the numerical simulation.

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SYSTEMS WITH MEMORY, NONLOCALITY AND ANTICIPATION. SOME NEW EXAMPLES AND NEW RESEARCH PROBLEMS

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Recently some new solutions have been found for distributed media : dynamical chaos, oscillations, autowaves, synchronization and quite recently "chimera" states. Till now mostly the classical parabolic equations (or related with them) have been used. But now it was recognised that more accurate equations should be used (see for example [1]. Here we describe some possibilities for posing new research problems. First of all we consider the models with memory (relaxation). In such case on of the classes of models constitute the infinite systems of o.d.e of second order in time received by projection methods. Such systems remember the systems of coupled oscillators. So the problems of energy transitions on spectrumreceives new solutions (from large to small scales). At second, we consider quasilinear hyperbolic Burgers equation of second order in time [1]. However, it is not enough studied. There are examples of the new solutions in the report. Fractal theory is one of the most flourishing mathematic modeling directions which find its application in the use of new and new fields of technology and basic research, including cellular automata theory, pattern recognition, artificial intelligence etc. One of the most important its question is a determination and an estimation for fractal dimensions of such fractal sets. Here we consider the attractors of dynamic systems with multi-valued evolution operators. We have gotten an upper estimation for Hausdorff dimension of the attractors of such kind of systems. The basic example is the logistic (Ferhulst) equation with the anticipatory property. Finally, the new possibilities supply the accounting of nonlocality. This follows to presumable origin of new "chimera" states in hydrodynamics. In addition, accounting of anticipation follows to the possibilities of multivalued "chimera" states.

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ONE SPECIAL CONSTRUCTION IN THE SPECTRAL THEORY OF C_0-SEMIGROUPS

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In joint work with Prof. Dr. Grigory M. Sklyar (Institute of Mathematics, University of Szczecin, Szczecin, Poland & Kharkiv National University of V. N. Karazin, Kharkiv, Ukraine) we present the construction of the generator of a C_0-group with the following properties. It has simple eigenvalues $\{\lambda_n\}_{n=1}^{\infty}$, which essentially cluster at infinity, i.e. $\lim\limits_{n\to\infty} |\lambda_n - \lambda_{n+1}| = 0$, and corresponding family of eigenvectors is dense but do not form a Schauder basis. This construction is closely related with the recent results of G.Q. Xu et al. [1] and H. Zwart [2] on Riesz basis property of eigenvectors (eigenspaces) of infinitesimal operators. The discrete Hardy inequality for $p = 2$ plays a key role in our approach.

Let H be a Hilbert space with norm $\|\cdot\|$ and Riesz basis $\{e_n\}_{n=1}^{\infty}$. Consider the operator T defined on H as $Te_n = e_{n+1}$. By $H_1(\{e_n\})$ we then denote the completion of the space $H_1^0(\{e_n\}) = \{x \in H : \|x\|_1 = \|(I-T)x\|\}$. It can be shown that

$$H_1(\{e_n\}) = \left\{ x = (\mathbf{f}) \sum_{n=1}^{\infty} c_n e_n : \{c_n\}_{n=1}^{\infty} \in \ell_2(\Delta) \right\},$$

where $(\mathbf{f}) \sum\limits_{n=1}^{\infty} c_n e_n$ is a formal series, $\ell_2(\Delta)$ is the space of all sequences whose differences are 2-absolutely summable, and Δ denotes a difference operator. It turns out that $\{e_n\}_{n=1}^{\infty}$ is dense in $H_1(\{e_n\})$ but does not form a Schauder basis in $H_1(\{e_n\})$.

The main result of the work can be formulated as follows. The operator $A : H_1(\{e_n\}) \supset D(A) \mapsto H_1(\{e_n\})$ defined by the formula $Ax = A(\mathbf{f}) \sum\limits_{n=1}^{\infty} c_n e_n = (\mathbf{f}) \sum\limits_{n=1}^{\infty} i \ln n \cdot c_n e_n$, with domain

$$D(A) = \left\{ x = (\mathbf{f}) \sum_{n=1}^{\infty} c_n e_n \in H_1(\{e_n\}) : \{\ln n \cdot c_n\}_{n=1}^{\infty} \in \ell_2(\Delta) \right\},$$

generates a C_0-group $\{e^{At}\}_{-\infty<t<\infty}$ on $H_1(\{e_n\})$. Moreover, we note that, surprisingly, the constructed C_0-group $\{e^{At}\}_{-\infty<t<\infty}$ has a linear growth when $t \to \pm\infty$.

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DYNAMICAL SYSTEMS APPROACH FOR MULTIDIMENSIONAL PHASE TRANSFORMATION MODELS AND THEIR APPLICATIONS

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Coupled dynamic systems of partial differential equations (PDEs) provide a foundation for many application areas in science and engineering [1, 2]. Mathematical studies of such systems lead to very challenging problems, in particular when we have to deal with system nonlinearities in time-dependent situations. Associated challenges are amplified further when multidimensional problems have to be addressed.

In this contribution, we analyze a large class of multidimensional coupled nonlinear systems of PDEs describing phase transformations. Mathematically they can be cast in the dynamical systems framework. One of the most important consequences of that is a possibility of developing efficient reduction procedures for these multidimensional models to low dimensions where the dynamics can be analyzed and dealt with on low dimensional manifolds. However, in these cases traditional procedures representing all effects at leading order of a small parameter can result in misleading outputs. Our exemplifications here are based on mathematical models describing coupled nonlinear phenomena in materials with memory, where we focus on cubic-to-tetragonal martensitic phase transformations in three dimensional settings under dynamic loading conditions. Mathematically, the resulting models can be formulated as free boundary problems due to interfacial conditions between different phases of the material. Within the Landau framework of phase transformations based on non-monotone free energy functions, the systems of interest here are reducible to parabolic-hyperbolic equations and we discuss their mathematical treatments from both analytical and numerical perspectives, including our developed low dimensional reduction and isogeometric methodologies. A number of examples from applications, where such coupled dynamic models play an important role, are demonstrated and discussed in the context of these developed methodologies.

[1] R. V. N. Melnik and A. J. Roberts, *Modelling nonlinear dynamics of shape-memory-alloys with approximate models of coupled thermoelasticity*. Zeitschrift Fur Angewandte Mathematik und Mechanik (ZAMM) **83(2)** (2003), 93–104.

[2] R. Dhote, M. Fabrizio, R. Melnik, and J. Zu, *A three-dimensional non-isothermal Ginzburg-Landau phase-field model for shape memory alloys*. Modelling and Simulation in Materials Science and Engineering **22 (8)** (2014), 085011.

ON PHENOMENA CONNECTED WITH INFINITE IFS

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We establish several new probabilistic, fractal and number theoretical phenomena connected with the Q_∞-expansion which is generated by iterated function systems consisting of infinite similitudes with positive ratios q_i such that $\sum_{i=1}^{\infty} q_i = 1$. First of all we show that system of cylinders of this expansion is, generally speaking, not faithful, i.e., to determine the Hausdorff dimension of a set from the unit interval one is not restricted to consider only coverings consisting of the above mentioned cylinders. We prove sufficient conditions for the non-faithfulness of the family of Q_∞-cylinders. On the other hand, sufficient conditions for the faithfulness of such covering systems are also found.

DETERMINISTIC DIFFUSION

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One-dimensional dynamical systems on the entire axis with discrete time are defined by the recurrence relation

$$x_{n+1} = f(x_n). \tag{1}$$

If a probability measure μ_0 (a normalized measure for which the measure of the entire axis is equal to 1) with density $\rho_0 : \mu_0(A) = \int\limits_A \rho_0(x)\,dx$ is given at the initial time, then, for a unit of time, system (1) maps this measure into $\mu_1 : \mu_1(A) = \mu(f^{-1}(A))$, where $f^{-1}(A)$ is a complete preimage of the set A under the map f. The operator mapping the measure μ_0 into the measure μ_1 is called a Perron–Frobenius operator $\mathcal{F}$.

The dynamical system (1) with a function f satisfying property

$$f(k + x) = k + f(x),\ |x| < \frac{1}{2},\ k \in Z \tag{2}$$

is called a Lifted Dynamical System (LDS).

We say that the LDS (1)–(2) has a deterministic diffusion(DD) if, for any initial probability measure μ_0 with bounded density, there exists a sequence of numbers $\sigma_n^2 > 0$ and ξ_n and 1–periodic function $\alpha(x) \geq 0$, $\int\limits_{-1/2}^{1/2} \alpha(x)\,dx = 1$, such that the sequence of measures $\mu_n = \mathcal{F}^n \mu_0$ obtained from the initial measure by the n-fold action of the LDS, is asymptotically equivalent, as $n \to \infty$ to a sequence of normal measures with densities

$$\rho_n(x) = \frac{\alpha(x)}{\sigma_n\sqrt{2\pi}} e^{-\frac{(x-\xi_n)^2}{2\sigma_n^2}}.$$

We find conditions of existence of DD in LDS (1)–(2). We give exact values of coefficients of DD for the case of a linear function f in the main interval $I_0 = [-\frac{1}{2}, \frac{1}{2})$. Models are suggested in [2], including two-dimensional dynamical systems of the form $x_{n+1} - x_n = x_n - x_{n-1} + f(x_n)$. They are useful within consideration of point particles transport in a billiard channel with complex boundary. DD is anomalous in such systems.

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[2] S. Albeverio, G. Galperin, I. Nizhnik and L. Nizhnik, Regular and Chaotic Dynamics **10** (3) (2005), 285-306.

STUDIES ON CHAOS AND HYPERCHAOS IN FOUR-DIMENSIONAL SPROTT SYSTEMS

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Dynamical analysis of chaotic and hyperchaotic attractors have been carried out on generated four-dimensional (4D) Sprott systems in the present work. J.C. Sprott [1] published a catalogue of nineteen (19) simple three-dimensional (3D) chaotic systems where it was demonstrated that there could be extremely simple three-dimensional chaotic systems compared with the work of Lorenz [2] and Rossler [3] in terms of algebraic representation rather than referring to the physical processes being modeled.

Practically, 4D or higher modes are better models for dynamical systems (dimensions is the number of variables considered in the model) [4]. Rossler [5] proposed the first (4D) hyperchaotic attractor, since then a number of hyperchaotic attractors and techniques for their generation have been reported from a hitherto (3D) chaotic system numerically and experimentally such as the addition of a linear simple-state-feedback controller [6] and in an open-loop manner by sinusoidal parameter perturbations [7, 8]. Generation of hyperchaotic dynamics has relied on mixing bifurcation analysis and computer simulations since there is no unified method for the construction of chaotic and hyperchaotic systems [9].

Lyapunov exponents algorithm was used in the present work to derive 4D algebraically simple chaotic and hyperchaotic Sprott systems from the 3D algebraically simple systems. These set of seventeen (17) (out of the nineteen proposed by Sprott) 4D dissipative systems display simple chaotic and hyperchaotic dynamics with parameter perturbations in the systems. Dynamical analysis was carried out using Lyapunov exponents, bifurcation diagrams, Poincare maps and phase portraits to authenticate the existence of these attractors which have potential applications in secure communications, neural networks, complex biological systems and laser physics.

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INVARIANT MEASURES IN PROJECTIVE BUNDLE

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The aim of the talk is to analyze the connection between Lyapunov exponents and invariant measures in projective bundle by using the symbolic image technique. Symbolic image of a dynamical system with respect to a covering is a directed graph with vertices corresponding to cells of the covering and edges corresponding to transitions between cells by system dynamics. The transformation of the system flow into a symbolic image allows reducing the problems of dynamical systems to the tasks on graphs. In this case a flow on the graph corresponds to an invariant measure and a mean of the graph labeling corresponds to a Lyapunov exponent. Symbolic image is a tool which may be successfully applied both to prove important results and to perform computer modelling of complex dynamical systems. The Morse spectrum is a collection of exponents of all orbits. The implementation of symbolic image gives an opportunity to calculate the Morse spectrum and to check hyperbolicity in complicated cases. The example of such a verification is given.

TOPOLOGICALLY CONJUGATED UNIMODAL INTERVAL MAPS

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Let $v \in (0,\, 1)$ be and arbitrary real and let f and f_v be $[0,\, 1] \to [0,\, 1]$-maps, which are defined as follows:

$$f(x) = \begin{cases} 2x, & x \leqslant 1/2; \\ 2-2x, & x > 1/2, \end{cases} \qquad f_v(x) = \begin{cases} \dfrac{x}{v}, & x \leqslant v; \\ \dfrac{1-x}{1-v}, & x > v. \end{cases}$$

We will pay attention to the following problems:

1. It is known [2], that mappings f and f_v are topologically conjugated and correspond homeomorphism h has the derivative 0 almost everywhere. We show, that for $v < 1/2$ the derivative h' equals $+\infty$ on the dense subset of $[0,\, 1]$;

2. We find evident formulas for the mapping h, which defines the topological conjugation of f and f_v, defining h as some convergence limit of functions;

3. The definition of topological conjugateness of f and f_v leads to the system of linear functional equations. The solution of each of these functional equations is given in [1] and each of them depends on an arbitrary function, which should be found from another equation. We study properties of these “arbitrary” functions;

4. Any continuous solution $h:\ [0,\, 1] \to [0,\, 1]$ of the functional equation $h(f) = f(h)$ is piecewise linear and is the following. y-coordinates of sharp points of h are equal to either 0 or 1 and absolute value of its tangent is a constant integer for all points where it exists; the tangent value of h can be equal to either 1, or an arbitrary even integer;

5. We study the topological conjugation of the map f and an arbitrary piecewise linear unimodal map $g:\ [0,\, 1] \to [0,\, 1]$, such that their topological conjugation is defined by a piecewise linear homeomorphism h. We prove, that in this case increasing part of g defines its decreasing part and also deceasing part of g defines its increasing part.

These results were obtained during the productive conversations with Dr. Volodymyr Vasyliovych Fedorenko.

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ONE-DIMENSIONAL DYNAMICAL SYSTEM GENERATED BY ONE CONTINUOUS TWISTED FUNCTION

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Let $1 < s$ is fixed integer and $A_s \equiv \{0, 1, \dots, s-1\}$ is the alphabet. The series

$$x = \sum_{k=1}^{\infty} \alpha_k s^{-k} \equiv \Delta^s_{\alpha_1\alpha_2\ldots\alpha_k\ldots},$$

is called s-adic representation of real number $x \in [0,1]$. Last short (sybolic) record $\Delta^s_{\alpha_1\alpha_2\ldots\alpha_k\ldots}$ is called s-adic image of real number x. Let vector

$$\overline{q} = (q_0, q_1, \dots, q_8) = \left(\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, \frac{1}{3}\right),$$

$$\beta_0 = 0, \quad \beta_k = q_0 + q_1 + \ldots + q_{k-1} = \beta_{k-1} + q_{k-1}, \quad k = \overline{1,9}$$

is defined. We consider dynamical system with phase space $[0,1] \times [0,1] \subset R^2$ and mapping $T = \{T_0, T_1, \dots, T_8\}$, where

$$T_i(x,y) = T_i(\Delta^9_{\alpha_1\alpha_2\ldots\alpha_k\ldots}, \Delta^{\overline{q}}_{\alpha_1\alpha_2\ldots\alpha_k\ldots}) = T(\Delta^9_{i\alpha_1\alpha_2\ldots\alpha_k\ldots}, \Delta^{\overline{q}}_{i\alpha_1\alpha_2\ldots\alpha_k\ldots}) = \left(\frac{1}{9}x + \frac{i}{9}, q_i y + \beta_i\right).$$

The attractor of this dynamical system is connected set, that is the graph of continuous nowhere monotone function and this function on any open interval $I \subset [0,1]$ assumes every value between its $\inf_I f(x)$ and $\sup_I f(x)$ a non-denumerable number of times where $\inf < \sup$ (excluding intervals of constancy). Function has the following analytic description

$$f(x) = f(\Delta^9_{\alpha_1\alpha_2\ldots\alpha_k\ldots}) = \beta_{\alpha_1(x)} + \sum_{k=1}^{\infty}\left(\beta_{\alpha_k(x)} \prod_{j=1}^{k-1} q_{\alpha_j(x)}\right) \equiv \Delta^{\overline{q}}_{\alpha_1\alpha_2\ldots\alpha_k\ldots}, \ \alpha_k \in A_9.$$

In the report we put full description of topological, metric and fractal properties of functions f.

ROBUST FEEDBACK SYNTHESIS FOR A DISTURBED CANONICAL SYSTEM

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The paper deals with the robust feedback synthesis of a bounded control for a system with an unknown perturbation. Namely, we consider the system of the form

$$\dot{x}_1 = (1 + p(t,x))x_2,\ \dot{x}_2 = (1 + r_2 p(t,x))x_3,\ \ldots,\ \dot{x}_{n-1} = (1 + r_{n-1}p(t,x))x_n,\ \dot{x}_n = u. \quad (1)$$

Here $t \geq 0$, $x \in R^n$ is a state $(n \geq 2)$, $u \in R$ is a control satisfying the constraint $|u| \leq 1$, r_i, $i = 2, \ldots, n-1$ are given numbers, and $p(t,x)$ is an *unknown* perturbation, which, however, satisfies the constraint $d_1 \leq p(t,x) \leq d_2$.

Our approach is based on the controllability function method created by V. I. Korobov in 1979 [1]. The *global robust feedback synthesis problem* is to construct a control of the form $u = u(x)$, $x \in R^n$, such that: (i) $|u(x)| \leq 1$; (ii) the trajectory $x(t)$ of the closed system, starting at an arbitrary initial point $x(0) = x_0 \in R^n$, ends at the origin at a finite time $T(x_0, p) < \infty$ for any admissible perturbation $d_1 \leq p(t,x) \leq d_2$; (iii) the control is independent of $p(t,x)$.

The goal of our work is to find the largest interval $[d_1; d_2]$ and to propose a constructive control algorithm.

Let

$$F^{-1} = \left(\frac{(-1)^{2n-i-j}}{(n-i)!(n-j)!(2n-i-j+1)(2n-i-j+2)} \right)_{i,j=1}^{n},$$

$$D(\Theta) = diag\left(\Theta^{-\frac{2n-2i+1}{2}}\right)_{i=1}^{n}, \quad F^1 = ((2n-i-j+2)f_{ij})_{i,j=1}^{n}, \quad S = F\tilde{R} + \tilde{R}^* F.$$

Theorem. *Let us choose* $0 < \gamma_1 < 1$, $\gamma_2 > 1$. *Put*

$$\tilde{d}_1^0 = 1/\lambda_{min}((F^1)^{-1}S), \quad \tilde{d}_2^0 = 1/\lambda_{max}((F^1)^{-1}S),$$

$$d_1^0 = \max\{(1-\gamma_1)\tilde{d}_1^0;\ (1-\gamma_2)\tilde{d}_2^0\}, \quad d_2^0 = \min\{(1-\gamma_1)\tilde{d}_2^0;\ (1-\gamma_2)\tilde{d}_1^0\}.$$

Let the controllability function $\Theta(x)$ *is a unique positive solution of equation*

$$2a_0\Theta = (D(\Theta)FD(\Theta)x, x), \quad x \neq 0, \quad \Theta(0) = 0, \quad 0 < a_0 \leq 2/f_{nn}.$$

Then for all d_1 *and* d_2 *such that* $d_1^0 < d_1 < d_2 < d_2^0$, *the control of the form*

$$u(x) = -\Theta^{-\frac{1}{2}}(x)\ FD(\Theta(x))x/2$$

solves the global robust feedback synthesis problem for system (1). Moreover, the trajectory of the closed-loop system, starting at any initial point $x(0) = x_0 \in R^n$, *ends at the point* $x(T) = 0$, *where the time of motion* $\Theta(x_0)/\gamma_2 \leq T(x_0, d_1, d_2) \leq \Theta(x_0)/\gamma_1$.

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DIFFERENCE EQUATIONS AND INFINITE-DIMENSIONAL DYNAMICAL SYSTEMS

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The nonlinear difference equations with continuous time

$$x(t+1) = f(x(t)), \qquad t \in R^+, \tag{$*$}$$

where $f : I \to I$ is a continuous map of the interval $I \subseteq R$, exhibit a surprising variety of solutions, up to quasi-random ones, that are indistinguishable at large time from stochastic processes. Here we have a situation wherein behavioral complexities of nonlinear one-dimensional maps intrude into the solutions of difference equations, and we can observe "purely nonlinear" features of solutions, in particular:

– asymptotic discontinuity (gradient catastrophe),

– fractal geometry of graphs (up to likeness to space-filling curves),

– unpredictability (impossibility of foretelling on large time scales),

– self-stochastization (obedience to certain probabilistic laws).

This talk is a brief presentation of the recently published (2014, December) book "Difference Equations with Continuous Argument" by E.Romanenko. The book summarizes the many years' research, carried out in the Institute of Mathematics, and gives the first complete exposition of the qualitative theory of equation $(*)$.

Analysis is based on going to the infinite-dimensional dynamical system generated by equation $(*)$ on its space of initial states. This method, which belongs among the most commonly used tools for the study of evolutionary problems, poses some problems when employing to equation $(*)$: In typical situations, the associated dynamical system has no attractor in its own phase space. Overcoming this obstacle and identifying peculiar features of solutions, which it causes, take a central place in the book.

These findings assume major importance because many boundary value problems for partial differential equations are reducible to difference equations with continuous time, and the latter give an elegant scenarios for onset of spatio-temporal chaos and reconstructions inside chaos, in particular:

– cascade emergence of coherent structures,

– chaotic mixing,

– intermittency.

Difference equations of the form $(*)$ seems to be very useful and efficient in modeling a variety of complex nonlinear processes, including turbulence.

SENSITIVITY OF THE INDUCED SYSTEMS ON AN INTERVAL

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We consider the dynamical systems $(C(I), f)$, where $C(I)$ is the family of all closed connected non-empty subsets of a closed interval I. In fact, $C(I)$ is a family of all intervals $[a, b]$ with $a \leq b$ and $a, b \in I$. The function f is an arbitrary mapping from I to itself, which is naturally extended to subsets as $f(A) = f(x)|x \in A$. In $C(I)$ we use the Hausdorff metrics d_H, which could be defined as $d_H([a, b], [c, d]) = \max\{|a - c|, |b - d|\}$ for any two intervals $[a, b]$ and $[c, d]$.

We study the sensitivity of the mentioned systems. Namely, we analyze, how notably the iterations $f^n([a, b])$ will change, if we modify the starting interval $[a, b]$ in a very slight way. The main result is the fact, according to which there is an interval $[a, b] \subset I$, which is an equicontinuous point for all mappings f^n. To prove this fact author used a theorem of Fedorenko, by which the sequence of the intervals $f^n([a, b])$ is asymptotically periodic unless all such intervals are disjoint [1].

[1] V. V. Fedorenko, *Asymptotical periodicity of the trajectories of an interval*, Ukrainian Mathematical Journal, **61** (2009), 854–858.

TOPOLOGICAL CLASSIFICATION OF LINEAR MAPPINGS

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We consider pairs of linear mappings $(\mathcal{A}, \mathcal{B})$ of the form

$$V \underset{\mathcal{B}}{\overset{\mathcal{A}}{=\!=\!=}} W \tag{1}$$

in which $=\!=$ is $\rightleftarrows$ or $\rightrightarrows$; V and W are finite dimensional unitary or Euclidean spaces. We say that (1) transforms to

$$V' \underset{\mathcal{B}'}{\overset{\mathcal{A}'}{=\!=\!=}} W'$$

(with the same orientation of arrows) by bijections $\varphi_1 : V \to V'$ and $\varphi_2 : W \to W'$ if

$$\begin{array}{lll} \mathcal{A}'\varphi_1 = \varphi_2\mathcal{A} \text{ and } \mathcal{B}'\varphi_2 = \varphi_1\mathcal{B} & \text{for the case } \rightleftarrows \\ \mathcal{A}'\varphi_1 = \varphi_2\mathcal{A} \text{ and } \mathcal{B}'\varphi_1 = \varphi_2\mathcal{B} & \text{for the case } \rightrightarrows \end{array}$$

We say that $(\mathcal{A}, \mathcal{B})$ and $(\mathcal{A}', \mathcal{B}')$ are *linearly equivalent* if φ_1 and φ_2 are linear bijections and *topologically equivalent* if φ_1 and φ_2 are homeomorphisms.

A pair of linear mappings $(\mathcal{A}, \mathcal{B})$ is *regular* if $\mathcal{A}$ and $\mathcal{B}$ are bijections, and *singular* otherwise. Each pair of linear mappings $(\mathcal{A}, \mathcal{B})$ possesses a regularizing decomposition in direct sum of the regular part and indecomposable singular pairs of linear mappings.

We obtained classification of pairs of linear mappings up to topological equivalence in [1] for the case $\rightrightarrows$ and in [2] for the case $\rightleftarrows$. We combine these results in the following theorem.

Theorem 1 *The pairs of linear mappings $(\mathcal{A}, \mathcal{B})$ and $(\mathcal{A}', \mathcal{B}')$ of the form (1) are topologically equivalent if and only if their regular parts are topologically equivalent and their indecomposable singular summands are linearly equivalent.*

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[2] T. V. Rybalkina, *Topological classification of pairs of counter linear maps.* Mat. Stud. **39** (2013), 21–28 (in Ukrainian).

ON POTENTIALITY OF SOME EVOLUTIONARY EQUATIONS WITH DEVIATING ARGUMENTS

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Definition. The operator $N : D(N) \subset U \to V$ is said to be potential on the set $D(N)$ relative to the bilinear form $\Phi : V \times U \to \mathbb{R}$, if there exists a functional $F_N : D(F_N) = D(N) \to \mathbb{R}$ such that $\delta F_N[u, h] = \Phi(N(u), h) \quad \forall u \in D(N), \quad \forall h \in D(N'_u)$.

Theorem. Suppose that $D_t^* = -D_t$ on $D(N'_u)$; then for operator

$$N(u) \equiv P_{2u,t}u_{tt} + P_{1u,t}u_t + Q(t, u) = 0_V,$$

$$u \in D(N) \subseteq U \subseteq V, \quad t \in [t_0, t_1] \subset \mathbb{R}; \quad u_t \equiv D_t u \equiv \frac{d}{dt}u, \quad u_{tt} \equiv \frac{d^2}{dt^2}u.$$

to be potential on $D(N)$ relative to bilinear form $\Phi(\cdot, \cdot) \equiv \int_{t_0}^{t_1} < \cdot, \cdot > \, dt : V \times U \to \mathbb{R}$ it is necessary and sufficient to have on $D(N'_u)$

$$P_{2u} - P_{2u}^* = 0, \tag{1}$$

$$P_{2u}^{*\prime}(\cdot; u_t) = 0, \tag{2}$$

$$-2\frac{\partial P_{2u}^*}{\partial t} + P_{1u}^* + P_{1u} = 0, \tag{4}$$

$$-\frac{\partial^2 P_{2u}^*}{\partial t^2} + \frac{\partial P_{1u}^*}{\partial t} + Q'_u - {Q'_u}^* = 0, \tag{5}$$

$$-\left(\frac{\partial P_{2u}^*}{\partial t}\right)'_u(\cdot; u_t) - \frac{\partial P_{2u}^{*\prime}}{\partial t}(\cdot; u_t) + P_{1u}^{*\prime}(\cdot; u_t) + P'_{1u}(u_t; \cdot) - [P'_{1u}(u_t; \cdot)]^* = 0, \tag{6}$$

$$P'_{2u}(u_{tt}; \cdot) - P_{2u}^{*\prime}(\cdot; u_{tt}) - [P'_{2u}(u_{tt}; \cdot)]^* = 0, \quad \forall u \in D(N), \quad \forall t \in [t_0, t_1]. \tag{7}$$

Theorem is applied for the construction of variational principles for the given differential-difference equations with partial derivatives.

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ON SINGULARITY OF DISTRIBUTION OF RANDOM VARIABLES WITH INDEPENDENT SYMBOLS OF OPPENHEIM EXPANSIONS

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Any real number $x \in (0,1)$ leads to the Oppenheim expansion

$$x \sim \frac{1}{d_1} + \frac{a_1}{b_1}\frac{1}{d_2} + \ldots + \frac{a_1 a_2 \cdot \ldots \cdot a_n}{b_1 b_2 \cdot \ldots \cdot b_n}\frac{1}{d_{n+1}} + \ldots$$

where $a_n = a_n(d_1,\ldots,d_n)$, $b_n = b_n(d_1,\ldots,d_n)$ are positive integers and the denominators d_n are determined by the algorithm:

$$x = x_1; \quad d_n = \left[\frac{1}{x_n}\right] + 1; \quad x_n = \frac{1}{d_n} + \frac{a_n}{b_n}x_{n+1},$$

and satisfy inequalities $d_{n+1} > \frac{a_n}{b_n}d_n(d_n-1)$ [1].

We call expansion the restricted Oppenheim expansion (ROE) of x if a_n and b_n depend only on the last denominator d_n and if the function $h_n(j) := \frac{a_n(j)}{b_n(j)}j(j-1)$ is integer valued.

Each of the cylinders of ROE-expansion can be uniquely rewritten in terms of the Difference ROE-expansion ($\overline{ROE}$): $\alpha_1 = d_1$; $\alpha_{k+1} = d_{k+1} - \frac{a_k}{b_k}d_k(d_k-1)$.

Let $\xi_1(x)$, $\xi_2(x)$, $\ldots$, $\xi_n(x)$, $\ldots$ be a sequence of independent random variables and $\xi = \Delta^{\overline{ROE}}_{\xi_1(x)\xi_2(x)\ldots\xi_n(x)\ldots}$ be a random variable with independent symbols of $\overline{ROE}$-expansion, $P\{\xi_k = i_0\} = p_{i_0 k}$.

Theorem. *If there exist a sequence l_k, such that $\forall x \in [0,1]$: $\frac{a_{k-1}}{b_{k-1}}\frac{b_k^2}{a_k^2}\frac{1}{d_{k-1}(d_{k-1}-1)} < l_k$ and $\sum_{k=1}^{\infty} l_k < +\infty$, then for any digit i_0 almost all (with respect to the Lebesgue measure) real numbers $x \in [0,1]$ contain symbol i_0 of $\overline{ROE}$-expansion only finitely many times and the probability measure μ_ξ is singular with respect to Lebesgue measure.*

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ABOUT THE FAITHFULNESS OF CANTOR SERIES EXPANSION CYLINDERS FAMILY FOR THE PACKING DIMENSION CALCULATION

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The report is devoted to finding conditions for the fine packing systems faithfulness with respect to packing dimension calculation. The packing dimension $\dim_P$ [2] is fractal dimension in some sense dual to the Hausdorff dimension $\dim_H$.

Let us fix some family Φ of balls from a metric space M.

Definition. *A ball family Φ is called faithful with respect to packing dimension calculation if* $\dim_P(E) = \dim_P(E, \Phi), \forall E \subset M$.

Examples of packing faithful families:

1. The family of s-adic cylinders;
2. The family of Q-cylinders;
3. The family of $\tilde{Q}$-cylinders if $\inf_{i,j} q_{ij} > 0$.

Theorem. *Let Φ be the family of all possible closed intervals (cylinders), generated by the Cantor series expansion of real numbers.*

Then the family Φ is faithful for the Packing dimension if and only if

$$\lim_{k\to\infty} \frac{\ln n_k}{\ln n_1 \cdot n_2 \cdot \ldots \cdot n_{k-1}} = 0.$$

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NONLINEAR ALMOST PERIODIC DIFFERENCE EQUATIONS

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Let E be the Banach space with norm $\|\cdot\|$. Denote by C^0 the Banach space of continuous and bounded functions $x = x(t)$, $t \in \mathbb{R}$, with values in E and with norm $\|x\|_{C^0} = \sup\limits_{t\in\mathbb{R}} \|x(t)\|$.

Define the shift operator $S_\tau : C^0 \to C^0$, $\tau \in \mathbb{R}$, by the formulae $(S_\tau x)(t) = x(t+\tau)$, $t \in \mathbb{R}$.

The function $x \in C^0$ is called *almost periodic* if the set $\overline{\{S_\tau x : \tau \in \mathbb{R}\}}$ is compact in C^0.

Let $\mathcal{K}$ be the set of compact sets $K \subset E$ and let $R(x)$ be the set $\{x(t) : t \in \mathbb{R}\}$. We denote by $\mathcal{D}_K$ the set of all elements of $x \in C^0$ for each of which $R(x) \subset K$.

The operator $H : C^0 \to C^0$ is called *almost periodic* if for every set $K \in \mathcal{K}$ and a sequence $(\tau_k)_{k\geqslant 1}$ of real numbers there exists a subsequence $(\tau_{k_l})_{l\geqslant 1}$, which

$$\lim_{l_1\to\infty,\ l_2\to\infty} \sup_{x\in\mathfrak{D}_K} \left\| S_{\tau_{l_1}} H S_{-\tau_{l_1}} x - S_{\tau_{l_2}} H S_{-\tau_{l_2}} x \right\|_{C^0} = 0.$$

Consider the almost periodic difference operator $F : C^0 \to C^0$ defined by the formulae $(Fx)(t) = G(t, x(t), x(t+\tau_1), \ldots, x(t+\tau_k))$, $t \in \mathbb{R}$, where $x \in C^0$, $k \in \mathbb{N}$, $\tau_1, \ldots, \tau_k \in \mathbb{R}$ and $G : \mathbb{R} \times E^{k+1} \to E$ is operator such that $\operatorname{diam} F(\mathbb{R} \times M_1 \times \ldots \times M_{k+1}) < +\infty$ for all bounded sets $M_1 \subset E, \ldots, M_{k+1} \subset E$. Consider the difference equation

$$Fx = 0. \tag{1}$$

Fix an arbitrary set $K \in \mathcal{K}$. Let $N(K)$ be the set of all solutions of equation (1), each of which $R(x) \subset \mathcal{K}$ and $\operatorname{diam} R(x) > 0$. Suppose that $N(K) \neq \emptyset$.

Fix an arbitrary element $x^* \in N(K)$. Let $r(x^*, K) = \sup\{\|x - y\| : x \in \overline{R(x^*)}, y \in K\}$. Also fix the arbitrary number $\varepsilon \in [0, r(x^*, K)]$. We denote by $\Omega(x^*, K, \varepsilon)$ the set of all elements of $y \in C^0$, each of which $R(y) \subset K$ and $\|y - x^*\|_{C^0} \geq \varepsilon$.

Theorem. *Let us suppose that* $K \in \mathcal{K}$, $z \in N(K)$ *and* $\inf\limits_{y\in\Omega(z,K,\varepsilon)} \|Fy\|_{C^0} > 0$ *for each* $\varepsilon \in (0, r(z, K))$. *Then solution* z *of equation* (1) *is almost periodic.*

[1] Slyusarchuk V. Yu. *Conditions of almost periodicity of bounded solutions of nonlinear difference equations with continuous argument*, Nelinijni Kolyvannya, **16**, 1 (2013), 118–124 [in Ukrainian].

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ON SOME PROBLEMS OF FAITHFUL COVERING FAMILY OF CYLINDERS GENERATED BY THE Q_∞-EXPANSION

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Let Φ be a covering system consisting of Q_∞-cylinders of $[0,1)$.

Let $\dim_H(E,\Phi)$ be a Hausdorff dimension of set $E \subset [0,1)$ with respect to covering system Φ.

A covering family Φ is said to be faithful if

$$\dim_H(E,\Phi) = \dim_H(E), \quad \forall E \subset [0,1].$$

$\{q_i\}_{i=1}^{\infty}$ be a sequence whith $\sum\limits_{i=1}^{\infty} q_i = 1$ and $\sum\limits_{i=1}^{\infty} q_i^\alpha < \infty$ for $\alpha > 0$. Denote:

$$S(\alpha,i) = \sum_{k=i+1}^{\infty} q_i^\alpha$$

$$V(\alpha,i) = \frac{S(\alpha,i)}{q_i^\alpha}$$

$$F(\alpha,\beta,i) = V(\alpha,i)\beta^i, \quad \beta \in (0,1)$$

Hypothesis. The family $\Phi(Q_\infty)$ of all possible cylinders of the Q_∞-partition of the interval $[0,1)$ is faithful if and only if

$$\lim_{i\to\infty} F(\alpha,\beta,i) = 0 \tag{1}$$

for $\forall \beta \in (0,1)$.

Counterexample. Consider the following sequence:

$$q_i = \begin{cases} \dfrac{K}{2^i} & when \quad i = 2n-1, \\ \dfrac{K}{3^i} & when \quad i = 2n, \end{cases}$$

where $K = \frac{24}{19}$.

Then the covering system generated by the Q_∞-expansion is faithful, but sequence doesn't satisfy condition (1) for all $\beta \in (0,1)$.

A SMOOTHLY LINEARIZABLE CIRCLE DIFFEOMORPHISM WITH BREAKS

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A circle diffeomorphism with breaks is an orientation-preserving circle homeomorphism that is piecewise C^2 together with its inverse. By Denjoy's theorem, the equality of irrational rotation numbers is necessary and sufficient for such mappings to be continuously conjugate. Two such mappings are called break equivalent, if there exists a continuous conjugacy between them that sends every break point of the first one into a break point of the second one with the same size of break (i.e. the ratio of left-hand derivative to right-hand one).

Obviously, the condition of break equivalence is necessary for C^1 conjugacy. In some cases that condition has been proved also sufficient [1, 2] (rigidity results). In many cases [3, 4] it was shown that non-break-equivalence implies that the conjugacy is a singular function. We call circle diffeomorphisms with breaks break equivalent in broad sense, if one of them can be adjusted by a piecewise C^2 conjugacy to became break equivalent to another.

For years, there stood a colloquial hypothesis that only break-equivalent in broad sense circle diffeomorphisms can be smoothly conjugate. We disprove it by constructing an example of a piece-wise linear circle homeomorphism with 4 non-trivial break points lying on different trajectories that is absolutely continuously linearizable (i.e. conjugate to a rigid rotation). In other words, its invariant measure has absolutely continuous density w. r. t. the Lebesgue measure. The rotation number for our example can be chosen either Diophantine or Liouvillean, but not of bounded type. Also, this is a first constructive example of a circle diffeomorphism, which linearization is absolutely continuous, but not piece-wise C^1, as it is non-differentiable on an everywhere dense set of points.

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FOUR DIMENSIONAL AND SINGULAR PERTURBATION SYSTEMS OF DIFFERENTIAL EQUATIONS AND TWO DIMENSIONAL DYNAMICAL SYSTEM WITH IMPULSE

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Definition 1. Special four dimensional and singular perturbation system of differential equations is

$$\begin{cases} \varepsilon \dot{\overrightarrow{x}} = \overrightarrow{f}(\overrightarrow{x}, \overrightarrow{y}) \\ \dot{\overrightarrow{y}} = \overrightarrow{g}(\overrightarrow{x}, \overrightarrow{y}) \end{cases}$$

where ε is small positive parameter, $\overrightarrow{x} \in R^2$, $\overrightarrow{y} \in R^2$, $\overrightarrow{f} \in C^1(D)$, $\overrightarrow{g} \in C(D)$, $D \in R^4$

Let topology of R^2 to be a topology which is generated by two dimensional Euclid metric.

Definition 2. Special two dimensional dynamical system with impulse is four objects (W, M, A, H) where W is two dimensional subset of R^2 with relative topology; M is one dimensional subset of R^2 with relative topology which is defined by equation $G(x_1, x_2) = 0$ and $M \in \partial W$; A is mapping M in W which is named as an impulse action(jump operator); H is mapping topological product $(W \setminus M) \times R^1$ in W where $H|_{S_i} = H_i$, $S_i = \{(x_1, x_2) : (-1)^{i+1} \cdot G(x_1, x_2) > 0\}$ $H_i(t, x_1^0, x_2^0)$; is solution of the Cauchy problem

$$\begin{cases} \dot{x_1} = f_1(x_1, x_2) \\ \dot{x_2} = f_2(x_1, x_2) \end{cases}$$

where $(x_1, x_2 \in S_i)$, $(x_1^0, x_2^0 \in S_i)$.

Dynamics of elements $w \in W$ is accomplished by the following algorithm: if $w \notin W$ then w is moved by R^1 group action H until W will not be on M. If $W \in M$ then w undergoes mapping of A. On the basis of the theory of singular perturbation system of differential equations [1], it was given the solution of a problem with association of the special four dimensional and singular perturbation system of differential equations to special two dimensional dynamical system with impulse. It was also considered some problems of associated two dimensional dynamical systems with impulse. Some examples are given.

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COMPARISON OF TWO STRATEGIES IN THE PROBLEM OF "CONQUEST" OF TERRITORY

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The theory of conflict dynamical systems developed in [1–3] is applied for construction of an abstract model in the problem of redistribution of resource space (territory) between two opponents.

Assume the conflict space $\Omega = [0,1]$ is structured in the sense that it consecutively divided into regions $\Omega = \bigcup_{i_1,\dots,i_k=1}^{2} \Omega_{i_1\dots i_k}$, $|\Omega_{i_1\dots i_k}| = \left(\frac{1}{2}\right)^k$, $k = 1, 2, \dots$.

Let opponents A and B are initially distributed along Ω according to matrixes $P = \{a_{ij}\}_{i=1,2;j=1,2,\dots}$ with $a_{1j} = \alpha$, $a_{2j} = 1-\alpha$, $0 < \alpha < 1/2$ and $R = \{b_{ij}\}_{i=1,2;j=1,2,\dots}$ with $b_{1j} = \beta$, $b_{2j} = 1-\beta$, $0 < \beta < 1$, respectively. The occupation probabilities for opponents A and B to be in each region $\Omega_{i_1\dots i_k}$ are defined by $p_{i_1\dots i_k} = \alpha^m(1-\alpha)^{k-m}$ and $r_{i_1\dots i_k} = \beta^m(1-\beta)^{k-m}$, resp., where m stands for a number of indexes $i_l = 1, 1 \le l \le k$ in $\Omega_{i_1\dots i_k}$. According to the theory [2], as a result of the conflict interaction, a region $\Omega_{i_1\dots i_k}$ will conquered by one of the opponents, say A, only if $p_{i_1\dots i_k} > r_{i_1\dots i_k}$. Each such region we denote as $\Omega^A_{i_1\dots i_k}$.

Theorem. *If* $1/2 - \alpha < |1/2 - \beta|$, *then* $\lim_{k\to\infty} \sum_{i_1,\dots,i_k=1}^{2} \left|\Omega^A_{i_1\dots i_k}\right| = 1$.

Therefore, if the initial distribution of opponent A along Ω is more uniform than in opponent B, then the Lebesgue measure of the territory controlled by the opponent A converges to 1 with $k \to \infty$. In turn, B completely loses its territory. Thus, in the conflict struggle the strategy of uniform distribution is optimal.

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ON FRACTAL PROPERTIES OF PROBABILITY MEASURES WITH INDEPENDENT $x - Q_\infty$-DIGITS

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The report is devoted to the development of DP-approach to study the fractal properties of spectra and minimal dimensional supports of probability measures from a family, which contains probability measures with independent digits of Q_∞-representation, $\widetilde{Q}_\infty$-representation, Lüroth expansions and their alternating modifications as special cases.

There are many papers devoted to the study of continuous transformations preserving the Hausdorff-Besicovich dimension (for example, [1]). It has been proven, in particular, that the problem of study of continuous DP-transformations of the unit interval is equivalent to the problem of study of DP-properties of continuous probability distribution functions. At the same time there are no papers on the DP-properties of transformations, for which the set of points of discontinuity is an everywhere dense set on some closed interval. Obviously, such transformations are dominating (in the sense of cardinality). On the other hand, it is helpful to study such transformations from the point of view of the development of methods for the calculation of the fractal dimensions and investigating fractal properties of probability distributions.

The main approach to the study of probability measures with independent $x - Q_\infty$-digits, which is represented in this talk, is that for a fixed stochastic vector Q_∞ and a fixed real number $x \in [0, 1]$ propose the consider the bijection

$$\varphi\left(\Delta^{Q_\infty}_{\alpha_1(z)\alpha_2(z)\ldots\alpha_k(z)\ldots}\right) = \Delta^{x-Q_\infty}_{\alpha_1(z)\alpha_2(z)\ldots\alpha_k(z)\ldots},$$

and study conditions under which φ preserves the Lebegue measure and the Hausdorff–Besicovitch dimension on the unit interval.

To investigate DP-properties of the bijection φ we study the problem related to the faithfulness of the covering systems connected with the above mentioned expansions for the calculation the Hausdorff-Besicovich dimension.

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ABSTRACTS

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