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Comparison of two strategies in the problem of "conquest" of territory

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The theory of conflict dynamical systems developed in [1, 2, 3] is applied for construction of an abstract model in the problem of redistribution of resource space (territory) between two opponents.

Assume the conflict space $\Omega = [0,1]$ is structured in the sense that it consecutively divided into regions $\Omega = \bigcup_{i_1,\ldots,i_k=1}^{2} \Omega_{i_1\ldots i_k}$, $|\Omega_{i_1\ldots i_k}| = \left(\frac{1}{2}\right)^k$, $k = 1, 2, \ldots,$.

Let opponents A and B are initially distributed along Ω according to matrixes $P = \{a_{ij}\}_{i=1,2;j=1,2,\ldots}$ with $a_{1j} = \alpha$, $a_{2j} = 1-\alpha$, $0 < \alpha < 1/2$ and $R = \{b_{ij}\}_{i=1,2;j=1,2,\ldots}$ with $b_{1j} = \beta$, $b_{2j} = 1-\beta$, $0 < \beta < 1$, respectively. The occupation probabilities for opponents A and B to be in each region $\Omega_{i_1\ldots i_k}$ are defined by $p_{i_1\ldots i_k} = \alpha^m(1-\alpha)^{k-m}$ and $r_{i_1\ldots i_k} = \beta^m(1-\beta)^{k-m}$, resp., where m stands for a number of indexes $i_l = 1, 1 \le l \le k$ in $\Omega_{i_1\ldots i_k}$. According to the theory [2], as a result of the conflict interaction, a region $\Omega_{i_1\ldots i_k}$ will conquered by one of the opponents, say A, only if $p_{i_1\ldots i_k} > r_{i_1\ldots i_k}$. Each such region we denote as $\Omega^A_{i_1\ldots i_k}$.

Theorem. If $1/2 - \alpha < |1/2 - \beta|$, then $\lim_{k\to\infty} \sum_{i_1,\ldots,i_k=1}^{2} \left|\Omega^A_{i_1\ldots i_k}\right| = 1$.

Therefore, if the initial distribution of opponent A along Ω is more uniform than in opponent B, then the Lebesgue measure of the territory controlled by the opponent A converges to 1 with $k \to \infty$. In turn, B completely loses its territory. Thus, in the conflict struggle the strategy of uniform distribution is optimal.

References

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[3] V. Koshmanenko, *Existence theorems of the ω-limit states for conflict dynamical systems*,, Methods Funct. Anal. Topology, **20** (2014), no. 4, 379-390.