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A smoothly linearizable circle diffeomorphism with breaks

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A circle diffeomorphism with breaks is an orientation-preserving circle homeomorphism that is piecewise C^2 together with its inverse. By Denjoy's theorem, the equality of irrational rotation numbers is necessary and sufficient for such mappings to be continuously conjugate. Two such mappings are called break equivalent, if there exists a continuous conjugacy between them that sends every break point of the first one into a break point of the second one with the same size of break (i.e. the ratio of left-hand derivative to right-hand one).

Obviously, the condition of break equivalence is necessary for C^1 conjugacy. In some cases that condition has been proved also sufficient [1, 2] (rigidity results). In many cases [3, 4] it was shown that non-break-equivalence implies that the conjugacy is a singular function. We call circle diffeomorphisms with breaks break equivalent in broad sense, if one of them can be adjusted by a piecewise C^2 conjugacy to became break equivalent to another.

For years, there stood a colloquial hypothesis that only break-equivalent in broad sense circle diffeomorphisms can be smoothly conjugate. We disprove it by constructing an example of a piece-wise linear circle homeomorphism with 4 non-trivial break points lying on different trajectories that is absolutely continuously linearizable (i.e. conjugate to a rigid rotation). In other words, its invariant measure has absolutely continuous density w. r. t. the Lebesgue measure. The rotation number for our example can be chosen either Diophantine or Liouvillean, but not of bounded type. Also, this is a first constructive example of a circle diffeomorphism, which linearization is absolutely continuous, but not piece-wise C^1, as it is non-differentiable on an everywhere dense set of points.

References

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