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ON SOME OPEN PROBLEMS OF FAITHFUL COVERING FAMILY OF CYLINDERS GENERATED BY THE Q_∞-EXPANSION

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Let Φ be a covering system consisting of Q_∞-cylinders of $[0,1)$.

Let $\dim_H(E,\Phi)$ be a Hausdorff dimension of set $E \subset [0,1)$ with respect to covering system Φ.

A covering family Φ is said to be faithful if

$$\dim_H(E,\Phi) = \dim_H(E), \quad \forall E \subset [0,1].$$

$\{q_i\}_{i=1}^{\infty}$ be a sequence whith $\sum\limits_{i=1}^{\infty} q_i = 1$ and $\sum\limits_{i=1}^{\infty} q_i^{\alpha} < \infty$ for $\alpha > 0$. Denote:

$$S(\alpha,i) = \sum_{k=i+1}^{\infty} q_i^{\alpha}$$

$$V(\alpha,i) = \frac{S(\alpha,i)}{q_i^{\alpha}}$$

$$F(\alpha,\beta,i) = V(\alpha,i)\beta^i, \quad \beta \in (0,1)$$

Hypothesis. The family $\Phi(Q_\infty)$ of all possible cylinders of the Q_∞-partition of the interval $[0,1)$ is faithful if and only if

$$\lim_{i\to\infty} F(\alpha,\beta,i) = 0 \qquad (1)$$

for $\forall \beta \in (0,1)$.

Counterexample. Consider the following sequence:

$$q_i = \begin{cases} \frac{K}{2^i} & when \quad i = 2n-1 \\ \frac{K}{3^i} & when \quad i = 2n \end{cases}$$

where $K = \frac{24}{19}$.

Then the covering system generated by the Q_∞-expansion is faithful, but sequence doesn't satisfy condition (1) for all $\beta \in (0,1)$.