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Nonlinear almost periodic difference equations

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Let E be the Banach space with norm $\|\cdot\|$. Denote by C^0 the Banach space of continuous and bounded functions $x = x(t)$, $t \in \mathbb{R}$, with values in E and with norm $\|x\|_{C^0} = \sup\limits_{t\in\mathbb{R}} \|x(t)\|$.

Define the shift operator $S_\tau : C^0 \to C^0$, $\tau \in \mathbb{R}$, by the formulae $(S_\tau x)(t) = x(t+\tau)$, $t \in \mathbb{R}$.

The function $x \in C^0$ is called *almost periodic* if the set $\overline{\{S_\tau x : \tau \in \mathbb{R}\}}$ is compact in C^0.

Let $\mathcal{K}$ be the set of compact sets $K \subset E$ and let $R(x)$ be the set $\{x(t) : t \in \mathbb{R}\}$. We denote by $\mathcal{D}_K$ the set of all elements of $x \in C^0$ for each of which $R(x) \subset K$.

The operator $H : C^0 \to C^0$ is called *almost periodic* if for every set $K \in \mathcal{K}$ and a sequence $(\tau_k)_{k\geqslant 1}$ of real numbers there exists a subsequence $(\tau_{k_l})_{l\geqslant 1}$, which

$$\lim_{l_1\to\infty,\ l_2\to\infty} \sup_{x\in\mathfrak{D}_K} \left\| S_{\tau_{l_1}} H S_{-\tau_{l_1}} x - S_{\tau_{l_2}} H S_{-\tau_{l_2}} x \right\|_{C^0} = 0.$$

Consider the almost periodic difference operator $F : C^0 \to C^0$ defined by the formulae $(Fx)(t) = G(t, x(t), x(t + \tau_1), \ldots, x(t + \tau_k))$, $t \in \mathbb{R}$, where $x \in C^0$, $k \in \mathbb{N}$, $\tau_1, \ldots, \tau_k \in \mathbb{R}$ and $G : \mathbb{R} \times E^{k+1} \to E$ is operator such that $\operatorname{diam} F(\mathbb{R} \times M_1 \times \ldots \times M_{k+1}) < +\infty$ for all bounded sets $M_1 \subset E, \ldots, M_{k+1} \subset E$. Consider the difference equation

$$Fx = 0. \qquad (1)$$

Fix an arbitrary set $K \in \mathcal{K}$. Let $N(K)$ be the set of all solutions of equation (1), each of which $R(x) \subset \mathcal{K}$ and $\operatorname{diam} R(x) > 0$. Suppose that $N(K) \neq \emptyset$.

Fix an arbitrary element $x^* \in N(K)$. Let $r(x^*, K) = \sup\{\|x - y\| : x \in \overline{R(x^*)}, y \in K\}$. Also fix the arbitrary number $\varepsilon \in [0, r(x^*, K)]$. We denote by $\Omega(x^*, K, \varepsilon)$ the set of all elements of $y \in C^0$, each of which $R(y) \subset K$ and $\|y - x^*\|_{C^0} \geq \varepsilon$.

Theorem. *Let us suppose that $K \in \mathcal{K}$, $z \in N(K)$ and $\inf\limits_{y\in\Omega(z,K,\varepsilon)} \|Fy\|_{C^0} > 0$ for each $\varepsilon \in (0, r(z, K))$. Then solution z of equation (1) is almost periodic.*

References

[1] Slyusarchuk V.Yu. *Conditions of almost periodicity of bounded solutions of nonlinear difference equations with continuous argument*, Nelinijni Kolyvannya, **16**, No. 1 (2013), 118–124 (in Ukrainian).

[2] Slyusarchuk V.Yu. *Conditions of almost periodicity of bounded solutions of nonlinear difference equations with discrete argument*, Nelinijni Kolyvannya, **16**, No. 3, **16** (2013), 416–425 (in Ukrainian).