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On singularity of distribution of random variables with independent symbols of Oppenheim expansions

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Any real number $x \in (0,1)$ leads to the Oppenheim expansion

$$x \sim \frac{1}{d_1} + \frac{a_1}{b_1}\frac{1}{d_2} + \ldots + \frac{a_1 a_2 \cdot \ldots \cdot a_n}{b_1 b_2 \cdot \ldots \cdot b_n}\frac{1}{d_{n+1}} + \ldots$$

where $a_n = a_n(d_1, \ldots, d_n)$, $b_n = b_n(d_1, \ldots, d_n)$ are positive integers and the denominators d_n are determined by the algorithm:

$$x = x_1; d_n = \left[\frac{1}{x_n}\right] + 1; x_n = \frac{1}{d_n} + \frac{a_n}{b_n}x_{n+1},$$

and satisfy inequalities $d_{n+1} > \frac{a_n}{b_n}d_n(d_n - 1)$[1].

We call expansion the restricted Oppenheim expansion (ROE) of x if a_n and b_n depend only on the last denominator d_n and if the function $h_n(j) := \frac{a_n(j)}{b_n(j)}j(j-1)$ is integer valued.

Each of the cylinders of ROE-expansion can be uniquely rewritten in terms of the Difference ROE-expansion ($\overline{ROE}$): $\alpha_1 = d_1$; $\alpha_{k+1} = d_{k+1} - \frac{a_k}{b_k}d_k(d_k - 1)$.

Let $\xi_1(x), \xi_2(x), ..., \xi_n(x)...$ be a sequence of independent random variables and $\xi = \Delta^{\overline{ROE}}_{\xi_1(x)\xi_2(x)...\xi_n(x)...}$ be a random variable with independent symbols of $\overline{ROE}$-expansion, $P\{\xi_k = i_0\} = p_{i_0 k}$.

Theorem If there exist a sequence l_k, such that $\forall x \in [0,1] : \frac{a_{k-1}}{b_{k-1}}\frac{b_k^2}{a_k^2}\frac{1}{d_{k-1}(d_{k-1}-1)} < l_k$ and $\sum_{k=1}^{\infty} l_k < +\infty$, then for any digit i_0 almost all (with respect to the Lebesgue measure) real numbers $x \in [0,1]$ contain symbol i_0 of $\overline{ROE}$-expansion only finitely many times and the probability measure μ_ξ is singular with respect to Lebesgue measure.

References

[1] Galambos J., The ergodic properties of the denominators in the Oppenheim expansion of real numbers into infinite series of rationals, Proc. Amer. Math. Soc., 59, 1976, 9-13.

[2] Torbin G. Pratsiovyta I., The singularity of random Ostrogradskyi series of the second kind, Probab.Th. Math.Stat, 2010, 60-68.