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On potentiality of some evolutionary equations with deviating arguments

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Definition. The operator $N : D(N) \subset U \to V$ is said to be potential on the set $D(N)$ relative to the bilinear form $\Phi : V \times U \to \mathbb{R}$, if there exists a functional $F_N : D(F_N) = D(N) \to \mathbb{R}$ such that $\delta F_N[u,h] = \Phi(N(u), h) \quad \forall u \in D(N), \quad \forall h \in D(N'_u)$.

Theorem. Suppose that $D_t^* = -D_t$ on $D(N'_u)$; then for operator

$$N(u) \equiv P_{2u,t}u_{tt} + P_{1u,t}u_t + Q(t,u) = 0_V,$$

$$u \in D(N) \subseteq U \subseteq V, \quad t \in [t_0, t_1] \subset \mathbb{R}; \quad u_t \equiv D_t u \equiv \frac{d}{dt}u, \quad u_{tt} \equiv \frac{d^2}{dt^2}u.$$

to be potential on $D(N)$ relative to bilinear form $\Phi(\cdot,\cdot) \equiv \int\limits_{t_0}^{t_1} < \cdot, \cdot > dt : V \times U \to \mathbb{R}$ it is necessary and sufficient to have on $D(N'_u)$

$$P_{2u} - P_{2u}^* = 0, \tag{1}$$

$$P_{2u}^{*\prime}(\cdot; u_t) = 0, \tag{2}$$

$$-2\frac{\partial P_{2u}^*}{\partial t} + P_{1u}^* + P_{1u} = 0, \tag{3}$$

$$-\frac{\partial^2 P_{2u}^*}{\partial t^2} + \frac{\partial P_{1u}^*}{\partial t} + Q'_u - {Q'_u}^* = 0, \tag{4}$$

$$-\left(\frac{\partial P_{2u}^*}{\partial t}\right)'_u(\cdot; u_t) - \frac{\partial P_{2u}^{*\prime}}{\partial t}(\cdot; u_t) + P_{1u}^{*\prime}(\cdot; u_t) + P'_{1u}(u_t; \cdot) - [P'_{1u}(u_t; \cdot)]^* = 0, \tag{5}$$

$$P'_{2u}(u_{tt}; \cdot) - P_{2u}^{*\prime}(\cdot; u_{tt}) - [P'_{2u}(u_{tt}; \cdot)]^* = 0, \quad \forall u \in D(N), \quad \forall t \in [t_0, t_1]. \tag{6}$$

Theorem is applied for the construction of variational principles for the given differential-difference equations with partial derivatives.

References

[1] V.M.Filippov, V.M.Savchin, S.A.Budochkina *On the existence of variational principlrs for differential-difference evolution equations.* Trudy MIAN **283** (2013), pp. 25-39 (in Russian).