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Robust feedback synthesis for a disturbed canonical system

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The paper deals with the robust feedback synthesis of a bounded control for a system with an unknown perturbation. Namely, we consider the system of the form

$$\dot{x}_1 = (1 + p(t,x))x_2,\ \dot{x}_2 = (1 + r_2 p(t,x))x_3,\ \ldots,\ \dot{x}_{n-1} = (1 + r_{n-1} p(t,x))x_n,\ \dot{x}_n = u. \quad (1)$$

Here $t \geq 0$, $x \in R^n$ is a state ($n \geq 2$), $u \in R$ is a control satisfying the constraint $|u| \leq 1$, r_i, $i = 2, \ldots, n-1$ are given numbers, and $p(t,x)$ is an *unknown* perturbation, which, however, satisfies the constraint $d_1 \leq p(t,x) \leq d_2$.

Our approach is based on the controllability function method created by V. I. Korobov in 1979 [1]. The *global robust feedback synthesis problem* is to construct a control of the form $u = u(x)$, $x \in R^n$, such that:
1) $|u(x)| \leq 1$; 2) the trajectory $x(t)$ of the closed system, starting at an arbitrary initial point $x(0) = x_0 \in R^n$, ends at the origin at a finite time $T(x_0, p) < \infty$ for any admissible perturbation $d_1 \leq p(t,x) \leq d_2$; 3) the control is independent of $p(t,x)$. The goal of our work is to find the largest interval $[d_1; d_2]$ and to propose a constructive control algorithm.

$$\text{Let} \quad F^{-1} = \left(\frac{(-1)^{2n-i-j}}{(n-i)!(n-j)!(2n-i-j+1)(2n-i-j+2)} \right)_{i,j=1}^{n},$$

$$D(\Theta) = diag\left(\Theta^{-\frac{2n-2i+1}{2}}\right)_{i=1}^{n}, \quad F^1 = ((2n-i-j+2)f_{ij})_{i,j=1}^{n}, \quad S = F\tilde{R} + \tilde{R}^* F.$$

Theorem. Let us choose $0 < \gamma_1 < 1$, $\gamma_2 > 1$. Put $\tilde{d}_1^0 = 1/\lambda_{min}((F^1)^{-1}S)$,
$\tilde{d}_2^0 = 1/\lambda_{max}((F^1)^{-1}S)$, $d_1^0 = \max\{(1-\gamma_1)\tilde{d}_1^0;\ (1-\gamma_2)\tilde{d}_2^0\}$, $d_2^0 = \min\{(1-\gamma_1)\tilde{d}_2^0;\ (1-\gamma_2)\tilde{d}_1^0\}$.
Let the controllability function $\Theta(x)$ is a unique positive solution of equation

$$2a_0\Theta = (D(\Theta)FD(\Theta)x, x), \quad x \neq 0, \quad \Theta(0) = 0, \quad 0 < a_0 \leq 2/f_{nn}.$$

Then for all d_1 and d_2 such that $d_1^0 < d_1 < d_2 < d_2^0$, the control of the form

$$u(x) = -\Theta^{-\frac{1}{2}}(x)\ FD(\Theta(x))x/2$$

solves the global robust feedback synthesis problem for system (1). Moreover, the trajectory of the closed-loop system, starting at any initial point $x(0) = x_0 \in R^n$, ends at the point $x(T) = 0$, where the time of motion $\Theta(x_0)/\gamma_2 \leq T(x_0, d_1, d_2) \leq \Theta(x_0)/\gamma_1$.

References

[1] V. I. Korobov, *Controllability function method (Russian).* R&C Dynamics, M.-Izhevsk (2007).