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Topologically conjugated unimodal interval maps

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Let $v \in (0, 1)$ be and arbitrary real and let f and f_v be $[0, 1] \to [0, 1]$-maps, which are defined as follows:

$$f(x) = \begin{cases} 2x, & x \leqslant 1/2; \\ 2 - 2x, & x > 1/2, \end{cases} \qquad f_v(x) = \begin{cases} \frac{x}{v}, & x \leqslant v; \\ \frac{1-x}{1-v}, & x > v. \end{cases}$$

We will pay attention to the following problems:

1. It is known [2], that mappings f and f_v are topologically conjugated and correspond homeomorphism h has the derivative 0 almost everywhere. We show, that for $v < 1/2$ the derivative h' equals $+\infty$ on the dense subset of $[0, 1]$;

2. We find evident formulas for the mapping h, which defines the topological conjugation of f and f_v, defining h as some convergence limit of functions;

3. The definition of topological conjugateness of f and f_v leads to the system of linear functional equations. The solution of each of these functional equations is given in [1] and each of them depends on an arbitrary function, which should be found from another equation. We study properties of these "arbitrary" functions;

4. Any continuous solution $h : [0, 1] \to [0, 1]$ of the functional equation $h(f) = f(h)$ is piecewise linear and is the following. y-coordinates of sharp points of h are equal to either 0 or 1 and absolute value of its tangent is a constant integer for all points where it exists; the tangent value of h can be equal to either 1, or an arbitrary even integer;

5. We study the topological conjugation of the map f and an arbitrary piecewise linear unimodal map $g : [0, 1] \to [0, 1]$, such that their topological conjugation is defined by a piecewise linear homeomorphism h. We prove, that in this case increasing part of g defines its decreasing part and also deceasing part of g defines its increasing part.

These results were obtained during the productive conversations with dr. Volodymyr Vasyliovych Fedorenko.

References

[1] G.P. Peliukh, A. N. Sharkovskiy, *Introduction to functional equations theory.* Nukova Dumka, Kyiv, (1974).

[2] J. D. Skufca, E. M. Bolt, *A concept of homeomorphic defect for defining mostly conjugate dynamical systems.* Chaos, **03118** (2008), 1-18.