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Deterministic Diffusion

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One-dimensional dynamical systems on the entire axis with discrete time are defined by the recurrence relation

$$x_{n+1} = f(x_n). \tag{1}$$

If a probability measure μ_0 (a normalized measure for which the measure of the entire axis is equal to 1) with density $\rho_0 : \mu_0(A) = \int_A \rho_0(x)\,dx$ is given at the initial time, then, for a unit of time, system (1) maps this measure into $\mu_1 : \mu_1(A) = \mu(f^{-1}(A))$, where $f^{-1}(A)$ is a complete preimage of the set A under the map f. The operator mapping the measure μ_0 into the measure μ_1 is called a Perron–Frobenius operator $\mathcal{F}$.

The dynamical system (1) with a function f satisfying property

$$f(k + x) = k + f(x),\ |x| < \frac{1}{2},\ k \in Z \tag{2}$$

is called a Lifted Dynamical System (LDS).

We say that the LDS (1)–(2) has a deterministic diffusion(DD) if, for any initial probability measure μ_0 with bounded density, there exists a sequence of numbers $\sigma_n^2 > 0$ and ξ_n and 1–periodic function $\alpha(x) \geq 0$, $\int_{-1/2}^{1/2} \alpha(x)\,dx = 1$, such that the sequence of measures $\mu_n = \mathcal{F}^n \mu_0$ obtained from the initial measure by the n-fold action of the LDS, is asymptotically equivalent, as $n \to \infty$ to a sequence of normal measures with densities

$$\rho_n(x) = \frac{\alpha(x)}{\sigma_n \sqrt{2\pi}} e^{-\frac{(x-\xi_n)^2}{2\sigma_n^2}}.$$

We find conditions of existence of DD in LDS (1)–(2). We give exact values of coefficients of DD for the case of a linear function f in the main interval $I_0 = [-\frac{1}{2}, \frac{1}{2})$. Models are suggested in [2], including two-dimensional dynamical systems of the form $x_{n+1} - x_n = x_n - x_{n-1} + f(x_n)$. They are useful within consideration of point particles transport in a billiard channel with complex boundary. DD is anomalous in such systems.

References

[1] L. Nizhnik and I. Nizhnik, *Deterministic diffusion*, Preprint, (2015), http://arxiv.org/pdf/1501.00674.pdf[math.DS]

[2] S. Albeverio, G. Galperin, I. Nizhnik and L. Nizhnik, Regular and Chaotic Dynamics **10** (3) (2005), 285-306.