Dynamical Systems and Their Applications
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Dynamical Systems Approach for Multidimensional Phase Transformation Models and Their Applications

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Coupled dynamic systems of partial differential equations (PDEs) provide a foundation for many application areas in science and engineering [1, 2]. Mathematical studies of such systems lead to very challenging problems, in particular when we have to deal with system nonlinearities in time-dependent situations. Associated challenges are amplified further when multidimensional problems have to be addressed.

In this contribution, we analyze a large class of multidimensional coupled nonlinear systems of PDEs describing phase transformations. Mathematically they can be cast in the dynamical systems framework. One of the most important consequences of that is a possibility of developing efficient reduction procedures for these multidimensional models to low dimensions where the dynamics can be analyzed and dealt with on low dimensional manifolds. However, in these cases traditional procedures representing all effects at leading order of a small parameter can result in misleading outputs. Our exemplifications here are based on mathematical models describing coupled nonlinear phenomena in materials with memory, where we focus on cubic-to-tetragonal martensitic phase transformations in three dimensional settings under dynamic loading conditions. Mathematically, the resulting models can be formulated as free boundary problems due to interfacial conditions between different phases of the material. Within the Landau framework of phase transformations based on non-monotone free energy functions, the systems of interest here are reducible to parabolic-hyperbolic equations and we discuss their mathematical treatments from both analytical and numerical perspectives, including our developed low dimensional reduction and isogeometric methodologies. A number of examples from applications, where such coupled dynamic models play an important role, are demonstrated and discussed in the context of these developed methodologies.

References

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[2] R. Dhote, M. Fabrizio, R. Melnik, and J. Zu, *A three-dimensional non-isothermal Ginzburg-Landau phase-field model for shape memory alloys*. Modelling and Simulation in Materials Science and Engineering **22 (8)** (2014), 085011.