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Infinite orbit equivalence class for a minimal substitution dynamical system

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The seminal paper [2] answered, among other outstanding results, the question of orbit equivalence of *uniquely ergodic* minimal homeomorphisms of a Cantor set. It was proved that two such minimal systems, (X, T) and (Y, S), are orbit equivalent if and only if *the clopen values sets* $S(\mu) = \{\mu(E) : E \text{ clopen in X}\}$ and $S(\nu) = \{\nu(F) : F \text{ clopen in Y}\}$ coincide where μ and ν are the unique invariant measures with respect to T and S, respectively. Bratteli diagrams play an extremely important role in the study of homeomorphisms of Cantor sets because any minimal (and even aperiodic) homeomorphism of a Cantor set is conjugate to the Vershik map acting on the path space of a Bratteli diagram. This realization turns out to be useful in many cases, in particular, for the study of substitution dynamical systems because the corresponding Bratteli diagrams are of the simplest form.

In this talk, we focus on the study of orbit equivalence of minimal substitution dynamical systems. For any primitive proper substitution σ, we give explicit constructions of countably many pairwise non-isomorphic substitution dynamical systems $\{(X_{\zeta_n}, T_{\zeta_n})\}_{n=1}^{\infty}$ such that they all are (strong) orbit equivalent to (X_σ, T_σ). We show that the complexity of the substitution dynamical systems $\{(X_{\zeta_n}, T_{\zeta_n})\}$ is the essential difference that prevents them from being isomorphic.

Theorem. *Let σ be a proper substitution. Then there exist countably many proper substitutions $\{\zeta_n\}_{n=1}^{\infty}$ such that (X_σ, T_σ) is orbit equivalent to $(X_{\zeta_n}, T_{\zeta_n})$, but the systems $\{(X_{\zeta_n}, T_{\zeta_n})\}_{n=1}^{\infty}$ are pairwise non-isomorphic.*

Given a primitive (not necessarily proper) substitution τ, we find a stationary simple properly ordered Bratteli diagram with the least possible number of vertices such that the corresponding Bratteli-Vershik system is orbit equivalent to (X_τ, T_τ).

The results that will be presented during the talk are published in [1].

References

[1] S. Bezuglyi and O. Karpel, *Orbit Equivalent Substitution Dynamical Systems and Complexity.* Proc. Amer. Math. Soc. **142** (2014), 4155-4169.

[2] T. Giordano, I. Putnam, C. Skau, *Topological orbit equivalence and C^*-crossed products.* J. Reine Angew. Math. **469** (1995), 51-111.