ON FRACTAL PROPERTIES OF PROBABILITY MEASURES WITH INDEPENDENT $x - Q_\infty$-DIGITS

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The report is devoted to the development of DP-approach to study the fractal properties of spectra and minimal dimensional supports of probability measures from a family, which contains probability measures with independent digits of Q_∞-representation, $\tilde{Q}_\infty$-representation, Lüroth expansions and their alternating modifications as special cases.

There are many papers, devoted to the study of continuous transformations preserving the Hausdorff-Besicovich dimension (for example, [1]). It has been proven, in particular, that the problem of study of continuous DP-transformations of the unit interval is equivalent to the problem of study of DP-properties of continuous probability distribution functions. At the same time there are no papers on the DP-properties of transformations, for which the set of points of discontinuity is an everywhere dense set on some closed interval. Obviously, such transformations are dominating (in the sense of cardinality). On the other hand, it is helpful to study such transformations from the point of view of the development of methods for the calculation of the fractal dimensions and investigating fractal properties of probability distributions.

The main approach to the study of probability measures with independent $x - Q_\infty -$ digits, which is represented in this talk, is that for a fixed stochastic vector Q_∞ and a fixed real number $x \in [0,1]$ propose the consider the bijection

$$\varphi \left(\Delta^{Q_\infty}_{\alpha_1(z)\alpha_2(z)...\alpha_k(z)...} \right) = \Delta^{x - Q_\infty}_{\alpha_1(z)\alpha_2(z)...\alpha_k(z)...},$$

and study conditions under which φ preserves the Lebegue measure and the Hausdorff-Besicovitch dimension on the unit interval.

To investigate DP-properties of the bijection φ we study the problem related to the faithfulness of the covering systems connected with the above mentioned expansions for the calculation the Hausdorff-Besicovich dimension.

References

[1] S. Albeverio, M. Pratsiovytyi,G. Torbin, *Transformations preserving the Hausdorff-Besicovitch dimension.* Central European Journal of Mathematics **6**(2008), pp. 119-128.

[2] I. Garko, G. Torbin, *On $x - Q_\infty$-expansion of real numbers and problem connected with it.* International scientific conference: "Asymptotic methods in theory of differential equations" dedicated to the 80-th universary of Shkil M.I., NPU, Kyiv, 2012, P. 48.