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On Limit Cycle Bifurcations

Valery Gaiko
National Academy of Sciences of Belarus,
United Institute of Informatics Problems,
Minsk, Belarus
e-mail: valery.gaiko@gmail.com

We carry out the qualitative analysis of polynomial dynamical systems. To control all of their limit cycle bifurcations, especially, bifurcations of multiple limit cycles, it is necessary to know the properties and combine the effects of all of their rotation parameters. It can be done by means of the development of new bifurcational and topological methods based on the well-known Weierstrass preparation theorem and the Perko planar termination principle stating that the maximal one-parameter family of multiple limit cycles terminates either at a singular point which is typically of the same multiplicity (cyclicity) or on a separatrix cycle which is also typically of the same multiplicity (cyclicity) [1].

If we do not know the cyclicity of the termination points, then, applying canonical systems with field rotation parameters, we use geometric properties of the spirals filling the interior and exterior domains of limit cycles. Using this method, we have solved, e. g., the problem of the maximum number of limit cycles surrounding a singular point for an arbitrary polynomial system and Hilbert's Sixteenth Problem for a general Liénard polynomial system with an arbitrary (but finite) number of singular points [2]. Applying a similar approach, we have completed the strange attractor bifurcation scenario which connects globally the homoclinic, period-doubling, Andronov–Shilnikov, and period-halving bifurcations in the classical Lorenz system [3]. We discuss also how to apply this approach for studying global limit cycle bifurcations of discrete polynomial (and rational) dynamical systems which model the population dynamics in biomedical and ecological systems.

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References

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