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Application $(C)-$ propertias for Cesare summability methods $(C,1)$ of ergodic theorem

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Let the given sequence $S=\{S_n\in B: n\in N_0\}$, where B – Banach space, $N_0=N\cup\{0\}$. A closed convex set $G\subset B$ is the $(c)-$ set of sequences $S=\{S_n\}$, if $\forall\varepsilon>0\ \exists\delta(\varepsilon)>0$ $\exists([n_k(\varepsilon);m_k(\varepsilon)])$:

$$S_n\in G_\varepsilon\forall n\in[n_k(\varepsilon);m_k(\varepsilon)]\subset N\forall k\in N, \frac{m_k-n_k}{m_k}\geq\delta(\varepsilon),$$

where $G_\varepsilon-$ a closed convex $\varepsilon-$ neighborhood of a closed convex set $G\subset B$.

For the case $B=C$ of known methods Cesare $(C,\alpha),\alpha\geq 1$ $(c)-$ property: if a $\exists\lim_{n\to\infty}S_n=L,(C,\alpha)$ and closed convex set the $G\subset C$ is $(c)-$ set of sequences $S=\{S_n\}\subset C$, while $L\in G$ [1].

This property holds for the methods $(C,1)$ in the case of Banach spaces B.

Proposition. Let $(B,\Omega,\mu)-$ the space of normalized measure and $f\in L^1(B,\Omega,\mu)$. Then for each there $x\in B_e\subset B, \mu(B\backslash B_e)=0$ is a limit

$$lim_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n-1}f\left(T^kx\right)=\widehat{f}(x)\,[2,p.17-22],$$

that is, $lim_{n\to\infty}f(T^nx)=\widehat{f}(x)\,(C,1)$, and

1. $\widehat{f}(x)\in G_x\ \forall x\in B_e$, for which there is $(c)-$ a set $G_x\subset(B,\Omega,\mu)$ sequence $\{f(T^nx)\}$;
2. $lim_{n\to\infty}f(T^nx)=\widehat{f}(x)\,\forall x\in B_e$, in which each partial sequence boundary $\{f(T^nx)\}$ is the $(c)-$ point;
3. $x\in B\backslash B_e$, if for $x\in B$ there are two different $(c)-$ sets G_1 and G_2 sequences $\{f(T^nx)\}$ such that $G_1\subset(B,\Omega,\mu), G_2\subset(B,\Omega,\mu)$, $G_1\cap G_2=\emptyset$.

The statement can be formulated for the case $f(x)=\chi_A(x)$, where $\chi_A(x)-$ the indicator set $A\subset B$.

References

[1] Davydov N.A. One the methods of Cesaro summation series // Mat sat., 38 (80), 1956, p. 509-524.

[2] Kornfeld I. P., Sinai I. D., Fomin N. B. Ergodic theory. -M.: Science. Main editorial of physico-mathematical literature, 1980.