Dynamics of the Thue-Morse system of difference equations

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Abstract

The Thue-Morse system of difference equations was introduced in [1] as a model to understand the electric behavior (conductor or insulator) of an array of electrical punctual positive charges occupying positions following a one dimensional distribution of points called a *Thue-Morse chain* which it is connected to the sequence $\mathbf{t} = (0110100110010...)$ called also the *Thue-Morse sequence*. Unfolding the system of difference equations, we obtain the two-dimensional dynamical system in the plane given by

$$F(x, y) = (x(4 - x - y), xy)$$

The interest of such system was stated by A.Sharkovskĭi as an open problem and proposing some questions.

The most interesting dynamics of the system is developed inside an invariant plane triangle, where hyperbolic periodic points of almost all period appear, there are subsets of transitivity and invariant curves of spiral form around the unique inside fixed point.

In this talk we will present some results concerning the behavior of all points outside the triangle, completing the known dynamics of the system. In fact we have obtained that outside the triangle, the orbits of all points are unbounded. Some of them go to infinite in an oscillating way occupying the second and third quadrant of the plane and others are going in a monotone way to infinite. Outside the triangle there are no periodic points. Such new results has an interesting interpretation in terms of the physics of the problem. Additionally we will answer some of the questions stated by Sharkovskĭi concerning the inside of the mentioned triangle.

We will also present graphycal analysis of such evolutions and also the visualization of the dynamics of the system inside the triangle.

Additionally we will present results on another system associated to Fibonacci sequence whose unfolding in $\mathbf{R}^3$ is

$$F(x, y, z) = (y, z, yx - z)$$

References

[1] Y.Avishai and D.Berend, *Transmission through a Thue-Morse chain*, Physical Review B **45.6** (2011), 2717-2724.