Dynamical Systems and Their Applications
June 22 - 26, 2015, Kyiv, Ukraine

The cost of approximate controllability and an unique continuation result at initial time for the Ginzburg-Landau equation

Bianca-Elena Aramă

"Alexandru Ioan Cuza" University,
Iaşi, Romania
e-mail: bianca.gavrilei@yahoo.com

We consider the controlled Ginzburg-Landau equation with an internal distributed control in a sub-domain. The complex Ginzburg-Landau equation describes the evolution of a complex-valued field $y = y(x,t)$ by

$$\begin{cases} \partial_t y - (a+ib)\Delta y = Ry - (\alpha + i\beta)\left|y\right|^2 y + \chi_\omega u, & \text{where } t > 0,\ x \in \Omega \subset \mathbb{R}^N, \\ y = 0 & \text{on } \partial\Omega \times (0,T), \\ y(x,0) = y_0(x) & \text{in } \Omega. \end{cases} \tag{1}$$

Here a, b, R, α and β are some positive real numbers.

The fundamental technique approached in this paper is estimating Carleman type inequalities for the adjoint linearized system. We renew the computations made by Rosier and Zhang in [4], and obtain explicit coefficients in the Carleman estimates, with respect to T, where $[0,T]$ is the maximum interval of time we consider. We obtain explicit bounds of the cost of approximate controllability, i.e., of the minimal norm of a control needed to control the system approximately.

Given $y_0 \in L^2(\Omega)$, a final state $y_1 \in L^2(\Omega)$ and $\varepsilon > 0$, there exists a control $u \in L^2(\omega \times (0,T))$ such that the solution of (1) satisfies

$$\|y(T) - y_1\|_{L^2(\Omega)} \le \varepsilon.$$

As in [3], the approximate control u of minimal norm in $L^2(\omega \times (0,T))$ corresponding to $y_0 = 0$, $y_1 \in L^2(\Omega)$ and $\varepsilon > 0$ can be obtained by minimizing the convex functional J in $L^2(\Omega)$:

$$J(p_T) = 1/2 \iint\limits_{\omega\times(0,T)} \left|p\right|^2 dxdt + \varepsilon \left\|p_T\right\|_{L^2} - \int\limits_{\Omega} y_1 p_T dx.$$

We prove an unique continuation result at initial time, which relies on Carleman estimates with explicit coefficients. In [1], the authors establish an unique continuation result at initial time for a second-order parabolic operator P in $[0,T] \times \mathbb{R}^N$, $P = \partial_t + A.P = \partial_t + A$, where A is a second-order elliptic operator. In [2], Lefter and Lorenzi force the local result in [1], to a nonlocal one. The authors are interested when the unique continuation is global at initial time, i.e., when y solves the homogeneous parabolic equation in $L^2(0,T;L^2(\Omega))$ and

$\omega \subset\subset \Omega$ is an open subset, under which conditions on the behaviour of $\|y\|_{L^2(0,t;H^1(\omega))}$, when $t \longrightarrow 0$ one obtains that $y\,(x,0) = 0$ for all $x \in \Omega$.

We want to establish an unique continuation result at the initial time for the operator $P = \partial_t - (a+ib)\Delta$.

References

[1] P., Albano, D., Tătaru,*Unique continuation for second-order parabolic operators at the initial time*. Proc. Amer. Math. Soc. **132(4)** (2004), 1077–1085.

[2] C. G., Lefter, A., Lorenzi, *Approximate controllability for an integro-differential control problem*. Applicable Analysis (2008).

[3] J. L., Lions, *Remarks on approximate controllability*. J. Anal. Math. **59** (1992), 103–116.

[4] L., Rosier, B. Y., Zhang, *Null controllability of the complex Ginzburg-Landau equation*. Annales de L'Institut Henri Poincaré (2009).